GEOMETRICAL
DRAWING

PART. I.

WINTER

ELEMENTARY

GEOMETRICAL DRAWING.

PART 1.

INCLUDING PRACTICAL PLANE GEOMETRY;
THE CONSTRUCTION OF SCALES; THE USE OF THE SECTOR, THE
MARQUOIS SCALES, AND THE PROTRACTOR.

WITH A NUMEROUS COLLECTION OF EXERCISES

DESIGNED FOR THE USE OF

STUDENTS PREPARING FOR EXAMINATION.

BY

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CORRECTED AND ENLARGED.

TENTH EDITION.

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PREFACE.

THIS EDITION has been carefully corrected, and enlarged by the insertion of many fresh problems. The number of Examples has also been greatly increased by the introduction of papers set at the various Examinations mentioned in the Table of Contents.

SHOOTERS HILL: *May* 1873.

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ELEMENTARY

GEOMETRICAL DRAWING.

PART I.

INTRODUCTORY REMARKS.

BEFORE ATTEMPTING to draw the diagrams of the following problems, the beginner is advised to read and to pay particular attention to the following directions; his progress will thereby be facilitated, and he will speedily acquire the habits of neatness and accuracy, which are essential in this branch of study.

I. Use PAPER of good quality, not too highly glazed, and place it smoothly on the drawing-board.

II. Avoid as much as possible the use of INDIA-RUBBER on a drawing, previously to inking it in.

III. Use a tolerably hard and very fine PENCIL, and take care to keep it inclined at the same angle to the plane of the paper, throughout the operation of drawing a line, moving it in a plane perpendicular to that of the paper.

IV. Use INDIAN INK, free from grit, carefully rubbed down with water, introducing it between the blades of the pen, with a slip of paper, without wetting the outsides of the blades.

V. Keep all the INSTRUMENTS clean and in good order, being

especially careful to wipe the pens perfectly dry before laying them aside, and never to allow any ink to dry upon them.

VI. In solving problems, draw GIVEN LINES thin and continuous; LINES OF CONSTRUCTION, thin and dotted; and those lines which constitute the SOLUTION of the problem thick and continuous. Mark given points by small circles, as shown in the plates.

VII. Be very exact in determining the positions of points: when possible, avoid doing so by means of lines which intersect at an angle less than 20°. When a line has to be drawn through a point, ascertain, before drawing it, that the edge of the ruler is so placed that the point of the pencil will pass THROUGH the point.

VIII. In describing circles, do not let the stationary leg of the compasses make a hole through the paper.

IX. In drawing lines with the PEN, hold it in the same manner as that directed for tne pencil, so that both nibs may press equally upon the paper.

X. Do not pass over any problem until you perfectly UNDERSTAND its solution.

CHAPTER I.

PRACTICAL PLANE GEOMETRY.

Problem I.

To bisect a rectilineal angle.

Let M A N (Pl. I. Fig. 1) be the given angle.

With A as a centre, and a radius less than either A M or A N, describe a circle, cutting these lines in B and C.

With B and C as centres, and a radius greater than half the distance from B to C, describe two circles intersecting in D. Join A D.

A D will bisect the angle M A N.

To prove this, join B D, C D, and apply *Euc.* I. 8.

Obs. In Euclid's construction of this problem, the triangle B D C is equilateral, only because he has not previously shown how to construct an isosceles triangle.

Problem II.

At a point in a straight line, to make an angle equal to a given rectilineal angle.

Let M A N (Pl. I. Fig. 2) be the given angle, P the point, in the line P Q.

With A as a centre, and a radius less than either A M or A N, describe a circle cutting these lines in B and C.

With P as a centre, and a radius P Q equal to A B, describe the circle Q R.

With the dividers set off Q R equal to B C. Join P R.

The angle Q P R will be equal to the angle M A N.

For the arc Q R is equal to the arc C B, and (*Euc.* III. 27) in equal circles the angles which stand upon equal circumferences are equal to one another.

Obs. If the straight lines C B, Q R be drawn, the equality of the angles M A N, Q P R may be proved by *Euc.* I. 8.

PROBLEM III.

Through a point, to draw a straight line parallel to a given straight line.

Let M N (Pl. I. Fig. 3) be the given line, P the point.

With P as a centre describe a circle cutting M N in A.

With A as a centre, and a radius A P, describe a circle cutting M N in Q.

With the dividers make A R equal to P Q.

The straight line drawn through P and R will be parallel to M N (*Euc.* III. 27, and I. 27).

PROBLEM IV.

To divide a straight line into n equal parts; n being a power of 2.

Let M N (Pl. I. Fig. 4) be the given line.

1. Let $n=2$. With M and N as centres, and a radius greater than $\frac{1}{2}$ M N, describe two circles intersecting in A and B.

Join A B; if A B cut M N in a, a will be the point of bisection of M N (*Euc.* I. 8 and 4).

2. If $n=4$, bisect a N in b by a construction similar to that in the preceding case.

b N will be $\frac{1}{4}$ of M N.

In the same manner, by repeated bisections, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, &c., of the line M N may be found.

Problem V.

To draw a straight line, from a given point, perpendicular to a given straight line.

Let P be the given point, M N the line.

1. Let P (Pl. I. Fig. 5) be without and not near the end of M N.

With P as a centre describe a circle, cutting M N in A and B.

With A and B as centres, and a radius greater than $\frac{1}{2}$ A B, describe two circles intersecting in C.

Join P C; let it cut M N in Q.

P Q is the perpendicular required (*Euc.* I. 8, 4, and Def. 10).

2. Let P (Pl. I. Fig. 6) be in, and not near the end of, M N. Make P A equal to P B. With A and B as centres, and a radius greater than $\frac{1}{2}$ A B, describe two circles cutting in Q.

Join P Q; P Q will be the perpendicular required (*Euc.* I. 8).

3. Let P be without, and near the end of, M N (Pl. I. Fig. 7). In M N take a point A; join P A; if P A be not perpendicular to M N, bisect P A in C; with C as a centre and C A as a radius, describe the circle A Q P, cutting M N in Q.

Join P Q; P Q will be perpendicular to M N (*Euc.* III. 31).

4. Let P be in, and near the end of, M N (Pl. I. Fig. 8). Take a point C not in M N: join P C; if P C be not perpendicular to M N, with C as a centre, and radius C P, describe a circle cutting M N in A; draw the diameter A C Q.

Join Q P; P Q will be perpendicular to M N (*Euc.* III. 31).

Obs. The fourth case may be solved by the following construction:—

With P (Pl. I. Fig. 9) as a centre, and a radius of 4 equal parts, describe a circle cutting M N in A; and also a second circle, with a radius of 3 equal parts from the same scale.

With centre A and a radius of 5 such equal parts, describe a third circle cutting the second in Q.

Join P Q; P Q will be perpendicular to M N (*Euc.* I. 48). For, $3^2 + 4^2 = 5^2$.

N.B. This method is often employed in the field when no instrument for measuring angles is at hand.

Ex. Let it be required to start from the station P, on the line M N, in a direction at right angles to M N.

Measure the distance P A 8 feet, and the distances Q A, Q P, 10 and 6 feet respectively.

P Q will be perpendicular to M N, and is, therefore, the direction required.

It is evident that any other unit of length may be used instead of feet; also that the sides of the triangle may be any numbers which are to one another as 3, 4, and 5.

Problem VI.

To divide a straight line into n equal parts; n being any number whatever.

Let M N (Pl. I. Fig. 10) be the given line, and $n=13$. From M draw an indefinite straight line M A, perpendicular to M N.

With N as a centre, and a radius of thirteen equal parts, taken from a scale, and such that their sum is greater than M N, describe a circle cutting M A in A.

Join N A, and divide it into thirteen equal parts, by setting off along it thirteen times one of the equal parts taken from the scale.

Through the points of division g_1, g_2, g_3, g_4, &c. &c., draw g_1p_1, g_2p_2, g_3p_3, g_4p_4, &c. &c., parallel to M A. M N will be divided in the points p_1, p_2, p_3, p_4, &c., similarly to N A; but N A is divided into thirteen equal parts, therefore M N is also divided into thirteen equal parts.

Obs. This method of dividing a line will be found useful in the construction of plain scales.

Note. The line may, for ordinary purposes, be divided more simply by the following construction. Draw any line N A making an angle, about equal to half a right angle, with M N, set off on it with the dividers 13 equal parts, of any length as Ng_{12}, $g_{12}g_{11}$, &c.: join the last point A with M, and through all the other points g_1, g_2, &c. draw parallels to A M as g_1p_1, g_2p_2, &c.

Cor. To find any fraction of a given line.

Let M N (Pl. I. Fig. 10) be the given line, n the denominator of the fraction; in this case suppose $n=13$.

Describe the right-angled triangle M A N as before.

Complete the rectangle A B N M.

Through g_1, g_2, g_3, g_4, &c. &c., draw g_1m_1, g_2m_2, g_3m_3, g_4m_4, &c. &c., parallel to M N or to A B.

Then, by similar triangles—

$$Ng_{12} : g_{12}m_{12} :: AN : NM.$$

$$\text{but } N\,g_{12} = \tfrac{1}{13} \text{ of } A\,N \therefore g_{12}m_{12} = \tfrac{1}{13} \text{ of } M\,N.$$

Similarly it may be shown that $g_{11}m_{11} = \frac{2}{13}$ of M N; $g_{10}m_{10} = \frac{3}{13}$ of M N; $g_9m_9 = \frac{4}{13}$ of M N; &c. &c.

Obs. This will be applied in the construction of diagonal scales.

Problem VII.

To draw circles, of given radii, to touch each other.

Draw an indefinite straight line M N (Pl. I. Fig. 11); in it take a point P, as the point of contact.

Make P A, P D equal to the given radii.

With A and D as centres, A P, D P as radii, describe the circles Q P, S P. These circles will touch each other in P (*Euc.* III. 11).

In the same manner circles P V and P R may be described with C and B as centres, C P and B P as radii, touching each other, and Q P, S P, in the point P.

For all circles which pass through P, and have their centres in M N, touch each other in P (*Euc.* III. 11 and 12).

Problem VIII.

To draw a tangent to a given circle from a point either without or on the circumference.

1. Let the point P (Pl. I. Fig. 12) be without the circle Q C R.

Find S, the centre of the circle (*Euc.* III. 1).

Join P S, upon it, as a diameter, describe the semicircle P R S, cutting Q C R in R. Join P R; P R T will be the tangent required (*Euc.* III. 31 and 16).

2. Let the point Q (Pl. I. Fig. 12) be on the circumference.

Find the centre S, join Q S.

Through Q draw A Q B perpendicular to Q S (*Prob.* V. 4) A Q B will be a tangent (*Euc.* III. 16, cor.).

Cor. A circle may be described to touch a given straight line, as A B (Pl. I. Fig. 12) in a given point Q, by drawing from Q, Q S perpendicular to A B, making Q S equal to the radius of the required circle, and with centre S, radius S Q, describing the circle Q C R (*Euc.* III. 16).

PROBLEM IX.

Upon a straight line to describe a segment of a circle which shall contain a given angle.

Let M N (Pl. I. Fig. 13) be the given line, B the angle.

1. If B be a right angle, upon M N describe the semicircle M P N.

The angle in the segment M P N will be a right angle, and therefore equal to B (*Euc.* III. 31).

2. If B (Pl. I. Fig. 14) be not a right angle.

At the points M and N, in the line M N, make the angles N M D, M N D each equal to B.

Draw M A and N A, perpendicular to M D and N D respectively, intersecting in A; M A will be equal to N A (*Euc.* I. 6).

With A as a centre, and a radius A M, or A N, describe the circle M Q N P; the angle in the segment M Q N, or that in the segment M P N, will be equal to B (*Euc.* III. 32), accordingly as B is greater or less than a right angle.

PROBLEM X.

To describe a circle passing through three given points, not in the same straight line.

Let P, Q, R (Pl. I. Fig. 15) be the given points.

Join P Q, Q R, bisect the lines P Q, Q R in A and B.

Draw A C, B C perpendicular to P Q, Q R, respectively, and intersecting in C.

The point C being equidistant from P, Q, and R (*Euc.* I. 4), is consequently the centre of the circle P Q R, passing through the points P, Q, R (*Euc.* III. 9).

PROBLEM XI.

To find a mean proportional between two given lines.

Let A, B (Pl. I. Fig. 16) be the lines.

Draw an indefinite straight line M N; in it take a point P.

Make P D equal to A, P E equal to B.

Upon D E describe the semicircle D Q E.

Draw P Q perpendicular to M N, and meeting the circumference in Q.

P Q will be the line required (*Euc.* VI. 8).

Cor. To determine the side of a square whose area is given.

Let the area be n superficial units.

Take D P (Pl. I. Fig. 16) equal to n lineal units: and P E equal to one unit.

Determine P Q, the mean proportional between D P and P E: P Q will be the side of the square required: for

$$PQ^2 = PD \times PE = n \times 1 = n, \text{ and } PQ = \sqrt{n}.$$

Obs. This construction is useful when n is not a square number, and it is required to determine accurately the side of the square.

PROBLEM XII.

Through equidistant points in a straight line, to draw a series of parallel straight lines at a given distance apart.

Let M N (Pl. I. Fig. 17) be the given line; p_1, p_2, p_3, &c. &c., the given points; d the given distance.

On p_1p_2 describe a semicircle: in it place the straight line p_1m_2, equal to d:

Join p_2m_2, and produce p_1m_2 indefinitely.

Draw $p_3\, m_3$, $p_4\, m_4$, $p_5\, m_5$, &c. &c. parallel to $p_2\, m_2$; these will be the lines required (*Euc.* III. 31, and VI. 2).

Cor. The given distance *d* can never be greater than the distance between two of the given points.

Problem XIII.

To describe a circle of given radius touching two given straight lines which cut each other.

Let M N, N O (Pl. II. Fig. 4) be the given lines; bisect the angle M N O by the line N P (Prob. I.).

From any point in P N, draw B Q perpendicular to N O and equal to the radius of the circle.

Draw Q R parallel to N P, cutting N O in R.

Draw R C perpendicular to N O, cutting N P in C; C will be the centre of the circle.

Draw C S perpendicular to N M.

Then (*Euc.* I. 34) C R is equal to B Q; (*Euc.* I. 26) C R is equal to C S, therefore the circle described with C as a centre and radius C R will pass through S, and touch the lines M N and O N (*Euc.* III. 16).

Problem XIV.

To describe a circle, which shall touch a given line in a given point, and also touch a given circle.

Let M N (Pl. II. Fig. 5) be the given line, P the given point, C the centre of the given circle S T Q.

Draw P B perpendicular to M N; and C Q parallel to P B.

Join P Q; let P Q meet the given circle in T: join C T, and produce it to meet P B in O.

Then the triangles T C Q and P O T are similar; and C T is equal to C Q; therefore O P is equal to O T; also O P is perpendicular to M N; consequently the circle described with centre O and radius O P will fulfil the conditions (*Euc.* III. 16 and 12).

Problem XV.

To divide a straight line similarly to a given divided line.

Let M N (Pl. II. Fig. 6) be the line divided into any number of parts in p_1, p_2, p_3, p_4, &c. &c. P Q the line to be divided.

Draw M N parallel to P Q at a convenient distance from it.

Join M P, N Q; if M N be equal to P Q, M P will be parallel to N Q.

Draw $p_1\ a_1$, $p_2\ a_2$, $p_3\ a_3$, $p_4\ a_4$, &c. &c., parallel to M P. Then P Q will be divided in the points, a_1, a_2, a_3, a_4, &c. &c.; similarly to M N (*Euc.* I. 34).

If M P be not parallel to N Q (Pl. II. Fig. 7), let them meet in A. Join Ap_1, Ap_2, Ap_3, Ap_4, &c. &c., cutting P Q in the points a_1, a_2, a_3, a_4, &c. &c. Then P Q will be divided in these points similarly to M N (*Euc.* VI. 2).

Problem XVI.

To describe a parallelogram, equal to a given triangle, and having an angle equal to a given angle.

Let P Q R (Pl. II. Fig. 8) be the given triangle. Bisect any side Q R in S. Make the angle R S A equal to the given angle.

Through R draw R B parallel to S A; through P draw P A B parallel to Q R.

A B R S will be the parallelogram required (*Euc.* I. 41).

Cor. The construction shows how to bisect a triangle by a line drawn through one of its angles.

Problem XVII.

To describe a circle, which shall have a given radius, touch one given line, and have its centre in a straight line making a given angle with the former.

Let M N (Pl. II. Fig. 9) be the line which the circle is to touch; N P that which is to contain the centre.

From any point M in M N draw M Q perpendicular to M N, and equal to the given radius.

Draw Q C parallel to M N, and cutting P N in C; and C T parallel to Q M.

Then C T is equal to Q M (*Euc.* I. 34), and C T M is a right angle; therefore the circle whose centre is C and radius C T will touch M N (*Euc.* III. 16).

Problem XVIII.

To divide a straight line in extreme and mean ratio.

Let M N (Pl. II. Fig. 10) be the line.

Through M draw A C perpendicular to M N (*Prob.* V. 4).

Make M A equal to $\frac{1}{2}$ M N; A C equal to A N; M P equal to M C.

M N will be divided in the point P, so that N M : M P :: M P : P N (*Euc.* II. 11; VI. 17).

Problem XIX.

To reduce a rectilineal figure of *n* sides to an equivalent figure having a number of sides less than *n*.

Let the given figure A B C D E F G (Pl. II. Fig. 11) have seven sides. Produce A B indefinitely,

Join F A; through G draw G R parallel to F A.

Join F R; then (*Euc.* I. 37) the triangle F R A is equal to the triangle F G A;

The triangle F O A is common to both of these triangles;

Therefore the triangle F O G is equal to the triangle R O A. From the figure A B C D E F G take the triangle F O G; to the remainder add the triangle R O A; and the resulting figure, B C D E F R, will evidently be equal to the original figure, A B C D E F G, which has thus been reduced to an equivalent figure of six sides.

Join E R; through F draw F P parallel to E R.

Join E P; the figure B C D E P, of five sides, will be equal to the figure B C D E F R, and therefore equivalent to the figure A B C D E F G.

Join D B; through C draw C S parallel to D B; join D S; the figure P S D E, of four sides, will be equal to A B C D E F G.

Join E S; through D draw D Q parallel to E S.

Join E Q; the triangle E P Q will be equivalent to the original figure.

The proof in each step of the reduction is similar to that in the first.

Obs. This problem is of frequent occurrence when an irregular polygon of any number of sides has to be reduced to an equivalent triangle, for the purpose of calculating its area.

Ex. Draw E a perpendicular to P Q; then

the area of the triangle E P Q $= \frac{1}{2}$E $a \times$ P Q:

if, therefore, E a, P Q, be measured by means of a scale, the area may immediately be found.

Problem XX.

Upon a given line, to describe a rectilineal figure similar to a given rectilineal figure.

Let A B C D E F (Pl. II. Fig. 12) be the given figure; $a b$ the given line.

Join A C, A D, A E.

At the points a and b in the line $a b$ make the angles $b\,a\,c$, $a\,b\,c$, equal to B A C, A B C, respectively; at a and c in $a\,c$, make the angles $c\,a\,d$, $a\,c\,d$, equal to C A D, A C D; at a and d in $a\,d$, make the angles $e\,a\,d$, $a\,d\,e$, equal to E A D, A D E; at a and e in $a\,e$, make the angles $e\,a\,f$, $a\,e\,f$, equal to E A F, A E F.

The figure $a\,b\,c\,d\,e\,f$ will be similar to the figure A B C D E F.

Cor. A B C D E F : $a\,b\,c\,d\,e\,f$ :: A B^2 : $a\,b^2$ (*Euc.* VI. 20).

If, therefore, $a\,b\,c\,d\,e\,f$ is required to be $= \dfrac{\text{A B C D E F}}{n}$; $a\,b^2 = \dfrac{\text{A B}^2}{n}$, and $a\,b = \dfrac{\text{A B}}{\sqrt{n}}$. This shows how to describe a rectilineal figure similar to a given rectilineal figure, and having any ratio to it.

Problem XXI.

To determine the direction of the line which would bisect the angle contained by two straight lines intersecting beyond the limits of the drawing.

Let P Q and R S (Pl. III. Fig. 16) be the lines.

In P Q take a point O.

Draw O T parallel to S R.

Make O T equal to O P.

Join P T; produce it to meet S R in R.

Draw M N bisecting P R at right angles.

M N if produced, would pass through the point of intersection of P Q and R S; and bisect the angle between them (*Euc.* I. 5, 29 and 4).

Problem XXII.

Upon a given straight line, to describe an isosceles triangle having a given vertical angle.

Let P Q (Pl. III. Fig. 17) be the given line.

Produce Q P to S, and make the angle T P S equal to the given vertical angle.

Bisect the angle T P Q by the line P R.

Make the angle P Q R equal to the angle Q P R.

R P Q will be the triangle required (*Euc.* I. 32 and 13).

Problem XXIII.

To describe a square upon a given straight line.

Let A B (Pl. III. Fig. 18) be the line.

Draw A E at right angles to A B; make A F equal to A B; through F and B draw F D and B D respectively parallel to B A and A F. A F D B will be the square required (*Euc.* I. 46).

Obs. If A D be joined, A B : A D :: 1 : $\sqrt{2}$ (*Euc.* I. 47); this will be useful in describing one figure half or double another.

Problem XXIV.

To describe a square equal to the difference of two given squares.

Upon M N the side of the greater square describe a semicircle M P N (Pl. III. Fig. 19); in it place a line N P equal to the side of the less square (*Euc.* IV. 1).

Join M P, the square on M P will be equal to the difference of the squares on M N and N P (*Euc.* III. 31, and I. 47).

PROBLEM XXV.

To determine the position of a point at which lines, drawn from three given points, shall make with each other angles equal to given angles.

Let P, Q, R (Pl. III. Fig. 20) be the given points; on P R describe a segment of circle P S R, containing an angle equal to that which the lines drawn from P and R are to contain: complete the circle.

Make the angle P R T equal to the angle which the lines drawn from P and Q are to contain.

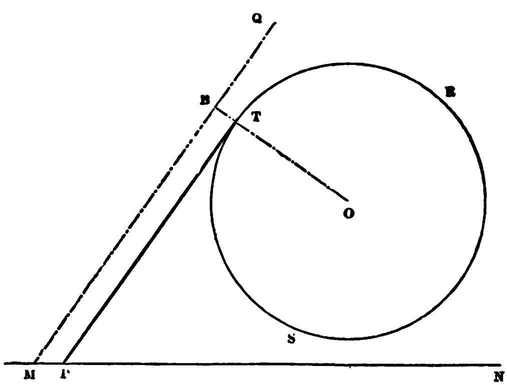
Join T Q; produce it to meet the circumference in S. S will be the required point; as will be seen by joining P S, R S (*Euc.* III. 21).

PROBLEM XXVI.

To draw a straight line which shall touch a given circle and make a given angle with a given line.

Let M N (Fig. 1) be the given line, O the centre of the given circle R S T.

FIG. 1.



From any point M in M N draw a straight line M Q, making with N M the angle N M Q, equal to the given angle (*Prob.* II.).

Draw O B perpendicular to M Q, and meeting the circumference in T (*Prob*. V. 1).

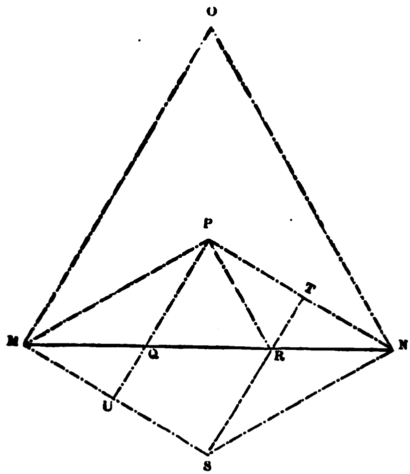
Draw T P parallel to Q M (*Prob*. III.). P T will be the line required (*Euc*. I. 29, and III. 16, Cor.).

PROBLEM XXVII.

To trisect a straight line.

Let M N (Fig. 2) be the given line.

FIG. 2.



On it describe the equilateral triangle M N O (*Euc*. I. 1).

Draw M P and N P, bisecting the angles O M N, O N M, respectively (*Prob*. I.).

Draw P Q and P R parallel to O M and O N (*Prob*. III.).

Then P Q R is an equilateral triangle (*Euc*. I. 29 and 32).

Also M Q is equal to P Q; and N R is equal to P R (*Euc*. I. 29 and 6); therefore, M Q = Q R = R N.

Otherwise. On M N as a diagonal, describe any parallelogram, M P N S.

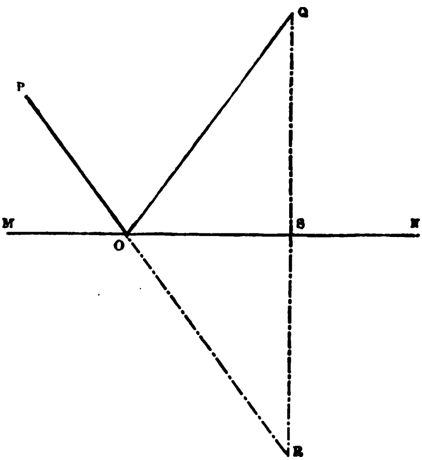
Draw P U and S T to the middle points of M S and P N, these lines will trisect M N in Q and R. (*Euc*. I. 34; VI. 2).

PROBLEM XXVIII.

From two points on the same side of a given line to draw two straight lines which shall meet in that line, and make equal angles with it.

Let M N (Fig. 3) be the line, P and Q the points.

FIG. 3.



Draw Q R perpendicular to M N, and cutting it in S (*Prob.* V. 1).

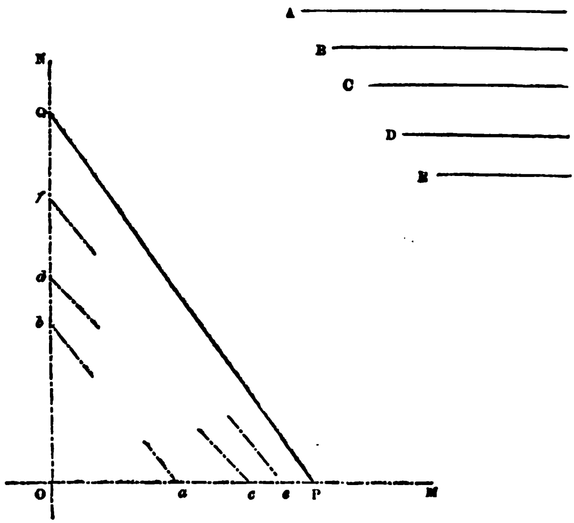
Make S R equal to Q S.

Join P R; let it cut M N in O. Join Q O. P O and Q O are the lines required.

For the angle Q O S is equal to the angle R O N (*Euc.* I. 4); therefore P O M=Q O N (*Euc.* I. 15, and Ax. 1).

O f = cd; O P = A; O Q = ef; join P Q; the square on P Q will be equal to the sum of the squares on A, B, C, D, E (*Euc.* I. 47).

FIG. 6.



PROBLEM XXXI.

To bisect a triangle by a line parallel to one of its sides.

Let A B C (Fig. 7) be the triangle. Bisect A C in D; draw D E perpendicular to A C and equal to A D: make A F equal to A E.

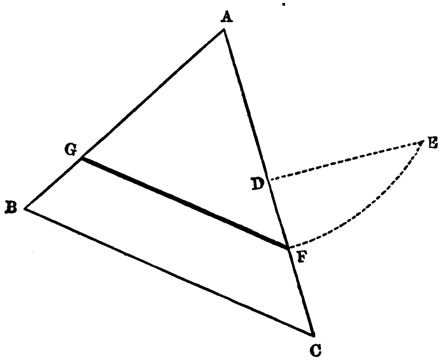
Through F draw F G parallel to B C. The triangle A G F will be half the triangle A B C.

$$\text{For A E : A C :: 1 : } \sqrt{2} \text{ (}\textit{Prob.}\text{ XXIII);}$$

$$\therefore \text{A F : A C :: 1 : } \sqrt{2}.$$

Also, triangle A G F : triangle A B C :: A F^2 : A C^2 (*Euc.* IV. 19) :: 1 : 2.

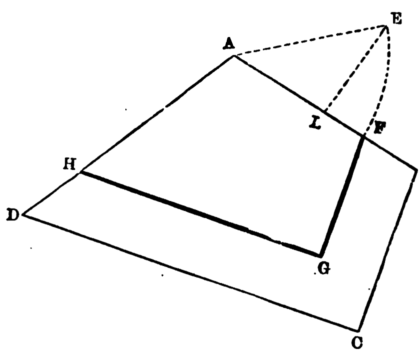
Fig. 7.



Problem XXXII.

To describe a polygon similar to a given polygon, but of half its area.

Fig. 8.



Let A B C D (Fig. 8) be the given figure. Find, as in *Prob.* XXXI., A F, so that A F : A B :: 1 : $\sqrt{2}$; on A F describe the figure A F G H, similar and similarly situated to A B C D

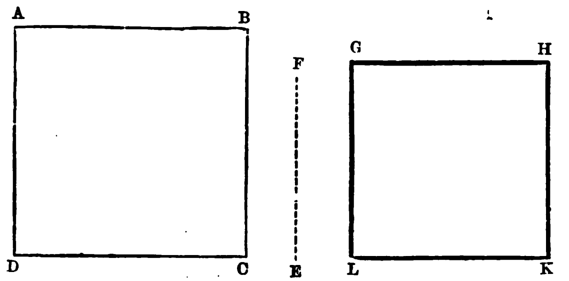
The figure A F G H will be half the figure A B C D (*Euc.* VI. 20).

Problem XXXIII.

To construct a square having a given ratio to a given square.

Let A B C D (Fig. 9) be the given square.

Fig. 9.



Take a straight line F E, such that D C : F E is the given ratio.

To D C and E F take a mean proportional L K.

On L K describe the square L K H G.

Then, $DC^2 : LK^2 :: DC : FE$ (*Euc.* v. *def.* 10).

Cor. A similar construction may be applied to find a rectilineal figure of any number of sides, similar to, and having a given ratio to, a given figure.

Problem XXXIV.

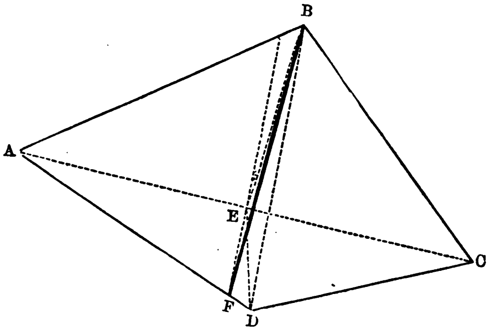
To bisect a trapezium by a straight line drawn through one of its angles.

Let A B C D (Fig. 10) be the given figure. Draw the diagonals A C, B D.

Bisect A C in E; through E draw E F parallel to B D, and

meeting A D in F; join B F; B F will bisect the figure A B C D.

FIG. 10.



Join B E, E D; B E D C is half the figure A B C D (*Euc.* I. 38), but B F D C is equal to B E D C (*Euc.* I. 37).

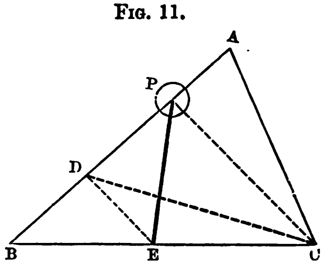
∴ B F D C is also half A B C D.

PROBLEM XXXV.

To divide a triangle into two parts, in a given ratio, by a straight line drawn through a given point in one of its sides.

Let A B C (Fig. 11) be the given triangle, P the point. Take a point D in A B, so that B D : D A is the given ratio.

FIG. 11.



Through D draw D E parallel to P C; join P E.

Then (*Euc.* I. 37) the triangle P B E is equal to the triangle B D C, and the triangle A D C is equal to the figure A P E C.

But (*Euc.* VI. 1) triangle B D C : triangle A D C :: B D : D A.

∴ Triangle B P E : figure A P E C :: B D : D A.

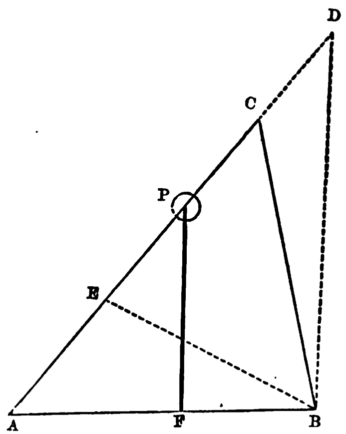
Problem XXXVI.

To divide a triangle into two parts, in a given ratio, by a line making a given angle with one of its sides.

Let A B C (Fig. 12) be the given triangle, A B D the given angle. Produce A C to meet B D in D.

Take the point E in A C so that A E : A C in the ratio of the part to be cut off to the whole triangle.

Fig. 12.



Take A P a mean proportional between A E and A D; draw P F parallel to B D; join E B.

The triangle A B D : triangle A F P :: $AD^2 : AP^2$,

:: A D : A E,

:: triangle A B D : triangle A B E;

∴ triangle A F P = triangle A B E.

But triangle A B E : triangle A B C :: A E : A C;

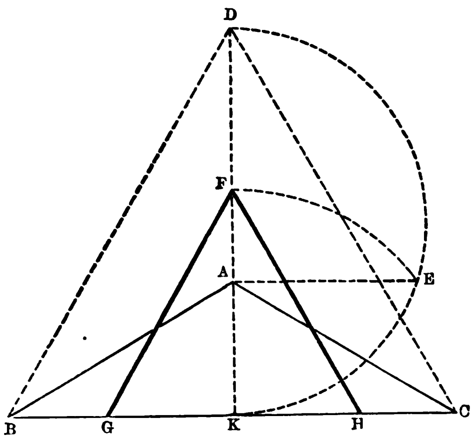
∴ triangle A F P : triangle A B C :: A E : A C.

Problem XXXVII.

To describe an equilateral triangle equal to a given isosceles triangle.

Let A B C (Fig. 13) be the given isosceles triangle, A B=A C. On B C describe the equilateral triangle B D C; join D A and produce it to K; D K will be at right angles to B C.

Fig. 13.



On D K describe the semicircle D E K; draw A E perpendicular to D K; make K F equal to K E; draw F H parallel to D C, F G parallel to D B.

The triangle F G H is equilateral because it is similar to B D C; also, K D : K F :: K F : K A (*Euc.* vi. 8),

but K D : K F :: K B : K G (*Euc.* vi. 2);

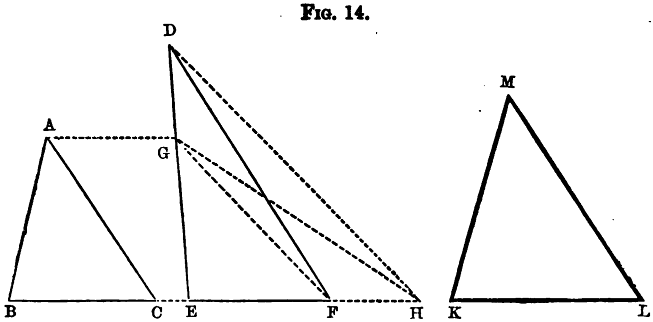
∴ K F : K A :: K B : K G, and the angle F K B is common to the triangles F K G, A K B; ∴ (*Euc.* vi. 15) the triangle F K G is equal to the triangle A K B, and consequently the triangle F G H is equal to the triangle A B E.

PROBLEM XXXVIII.

To describe a triangle equal to one given triangle and similar to another.

Let A B C (Fig. 14) be the triangle to which the required triangle is to be similar, D E F that to which it is to be equal; place the triangles so that B C and E F are in the same straight line; draw A G parallel to B F; join G F; draw D H parallel to G F, and meeting E F produced in H; join G H: the triangle

FIG. 14.



G E H is equal to D E F (*Euc.* I. 37): take K L a mean proportional to B C and E H; on K L describe the triangle K M L similar to A B C, so that B C and K L are homologous sides.
Then the triangle A B C : triangle G E H :: B C : E H;
∴ the triangle A B C : triangle D E F :: B C : E H,
:: B C^2 : K L^2.
But (*Euc.* VI. 19)
the triangle A B C : triangle K M L :: B C^2 : K L^2;
∴ the triangle K M L is equal to the triangle D E F.

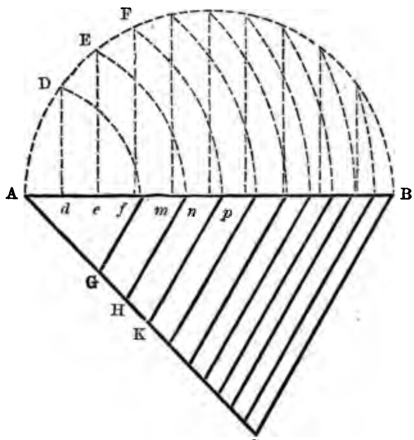
PROBLEM XXXIX.

To divide a triangle into *n* equal parts by lines parallel to its sides.

Let A B C (Fig. 15) be the given triangle. On A B describe

the semicircle A D B; divide A B into n equal parts in d, e, f, *etc.*; draw d D perpendicular to A B; make A m equal to A D; draw m G parallel to B C.

FIG. 15.



The triangle A m G : triangle A B C :: A m^2 : A B^2,

:: A d : A B,

:: 1 : n;

∴ triangle A m G is $\frac{1}{n}$ of triangle A B C.

Similarly it can be shown that A n H is $\frac{2}{n}$ of A B C, A p K $\frac{3}{n}$ of A B C, &c. &c.

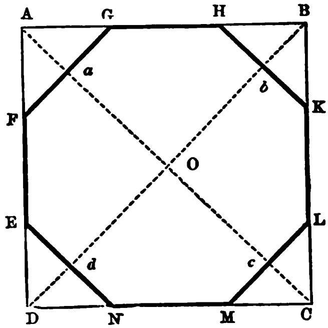
PROBLEM XL.

To place a regular octagon in a given square.

Let A B C D (Fig. 16) be the given square. Draw the diagonals A C, B D; make Oa=Ob=Oc=Od=half the side of

the square; draw G a F, E d N, M c L, K b H parallel to the diagonals respectively: E F G H K L M N will be the octagon

Fig. 16.

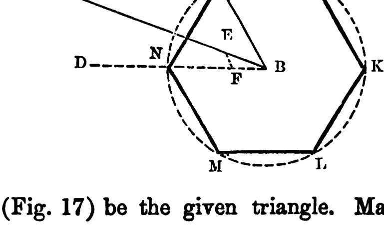


required. This may be shown at once by producing the sides which are parallel to the diagonals till they meet, when they will form a square equal to the given square.

Problem XLI.

To describe a regular polygon equal to a given triangle.

Fig. 17.



Let A B C (Fig. 17) be the given triangle. Make the angle

A B D equal to the angle subtended at the centre of the circumscribing circle by a side of the polygon.

Take B E$=\frac{1}{n}$ of B C (n being the number of sides of the required polygon); draw E F parallel to A B: then triangle A E B$=$A F B$=\frac{1}{n}$ of triangle A B C. Take B N a mean proportional to A B and B F; complete the triangle N B G by making B G$=$B N and joining G N.

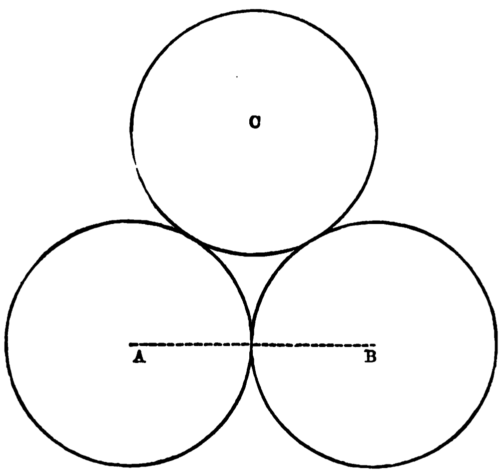
Then since A B : B N :: B G : B F, the triangle G B N$=$A B F; but A B F$=\frac{1}{n}$ of A B C.$\therefore$ the polygon when completed being n times G B N$=$the triangle A B C.

Problem XLII.

To describe three equal circles touching one another.

Let A B (Fig. 18) be equal to the diameter of each circle.

Fig. 18.



with A and B as centres and a radius A B describe two arcs intersecting in C.

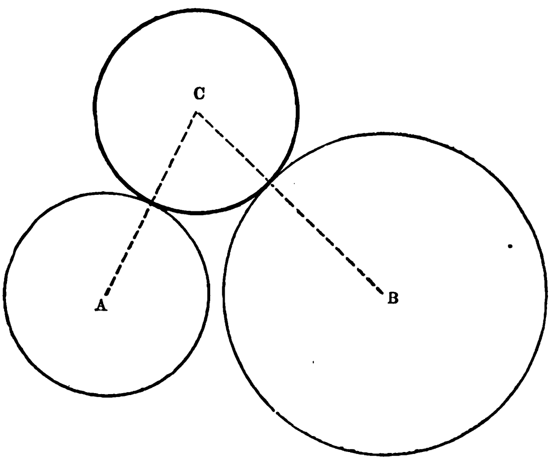
With A, B, and C as centres and a radius equal to half of A B describe three circles: these will be the circles required (*Euc.* III. 12).

Problem XLIII.

To describe a circle of given radius touching two given circles.

Let A and B (Fig. 19) be the centres of the given circles, A and B

Fig. 19.



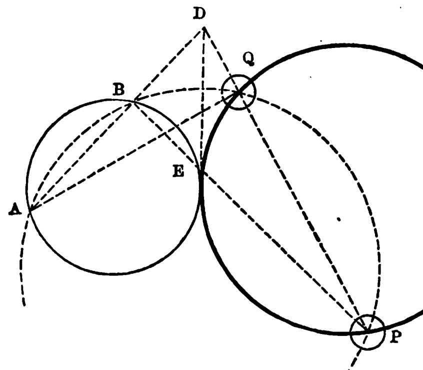
With A as a centre and a radius equal to the sum of tne radius of A and that of the circle required describe a circle; with centre B and a radius equal to the sum of the radius of B and that of the required circle describe a circle cutting the former in C; with C as a centre and the given radius describe a circle: this will touch A and B (*Euc.* III. 12).

PROBLEM XLIV.

To describe a circle which shall pass through two given points and touch a given circle.

Let P and Q (Fig. 20) be the given points, A B E the given circle.

FIG. 20.



Describe a circle A B Q P passing through P and Q and cutting the circle A B E in B and E; join A Q, P B; join A B and produce it to meet P Q produced in D: then the triangles A D Q and P D B are similar. Draw D E touching the circle A B E in E; describe a circle passing through P, Q and E: this will be the circle required.

For A D . D B$=$D E^2$=$P D . D Q, $\therefore$D E is a tangent to the circle F E P, which $\therefore$ touches A B E.

PROBLEM XLV.

To inscribe a square in the quadrant of a circle.

Let A B C (Fig. 21) be the given quadrant.

Bisect the angle B A C by the straight line A D meeting the

circumference in D; draw D E parallel to A C. and D F parallel to B A.

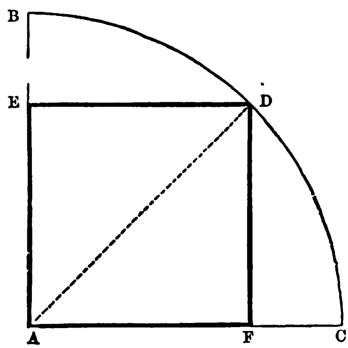


FIG. 21.

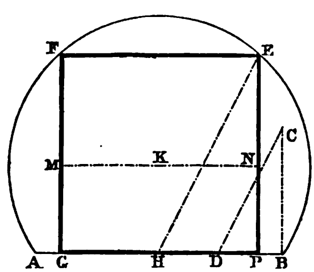
D E A F will be the square required (*Euc.* I. 34 and 6).

PROBLEM XLVI.

To describe a square in any segment of a circle.

Let A F E B (Fig. 22) be the given segment. Bisect A B in H; draw B C perpendicular to A B and equal to H B; bisect

FIG. 22.



H B in D; join C D; draw H E parallel to C D and E P parallel to C B; take H G equal to H P; draw G F perpendicular

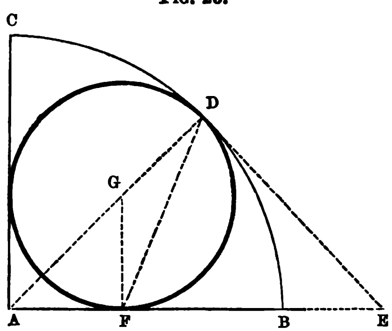
to A B; join F E: F G P E is the square required. Through K, the centre of the circle, draw M K N parallel A B.
Then (*Euc.* I. 34) M G=N P and (*Euc.* III. 14) M F=N E;
∴ F G=E P and F E=G P.
Also E P : H P :: C B : B D,
:: 2 : 1, ∴ E P=2 H P=G P.
And the figure is equilateral; it is rectangular by construction.

Problem XLVII.

To inscribe a circle in the quadrant of a circle.

Let A B C (Fig 23) be the given quadrant. Bisect the angle C A B by the straight line A D, meeting the circumference in

Fig. 23.



D; draw D E at right angles to A D and meeting A B produced in E; make E F equal to E D; join D F; draw F G perpendicular to A E and meeting A D in G.

Then the angle E D G=E F G, and the angle E D F= E F D; therefore the remainder G D F=G F D, and G D= G F (*Euc.* I. 6).

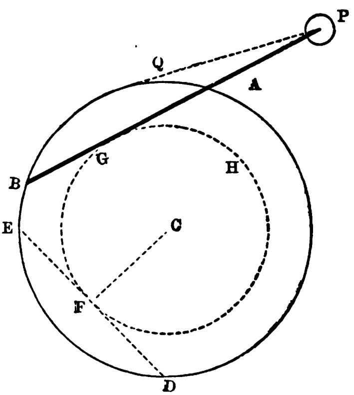
Therefore the circle whose centre is G and radius G D or G F is the circle required (*Euc.* III. 11 and 16).

PROBLEM XLVIII.

From a given point to draw a straight line which shall be cut in extreme and mean ratio by a given circle.

Let P (Fig. 24) be the given point, A B D the given circle, C its centre.

FIG. 24.



Draw P Q touching the circle A B D in Q; in this circle place the straight line E D equal to P Q; draw C F perpendicular to E D and describe the circle F G H. From P draw P A B touching this circle: P A B will be the line required.

For A B=E D (*Euc.* III. 14)=P Q,
and P B . P A=P Q^2 (*Euc.* III. 36)=A B^2;
$\therefore$ P B : A B : : A B : P A (*Euc.* VI. 17).

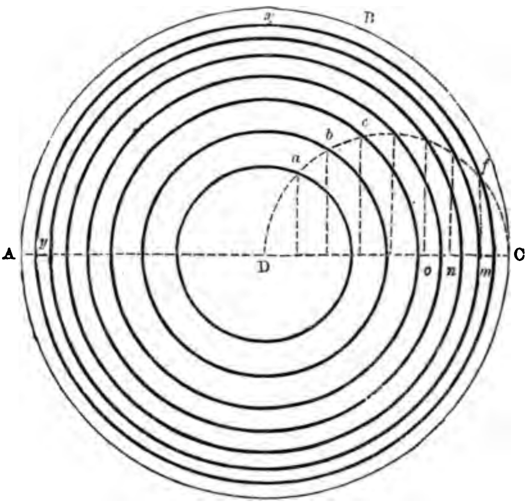
PROBLEM XLIX.

To divide a circle into n equal concentric annuli.

Let A B C (Fig. 25) be the given circle, D its centre. On C D describe the semicircle D a b C; take C m the nth part of D C;

draw $m\,f$ at right angles to C D and meeting D a b C in f; with D as a centre and a radius D f describe a circle $f\,x\,y$.

Fig. 25.



Then the circle A B C : circle $f\,x\,y$ :: D C² : D f^2
:: D C : D m;
∴ the circle A B C : ring A x B C :: D C : C m,
:: n : 1;

and the ring A n B C is $\frac{1}{n}$ of the circle A B C.

The other circles can be drawn in the same way by dividing D C into n equal parts as shown in the figure.

Problem L.

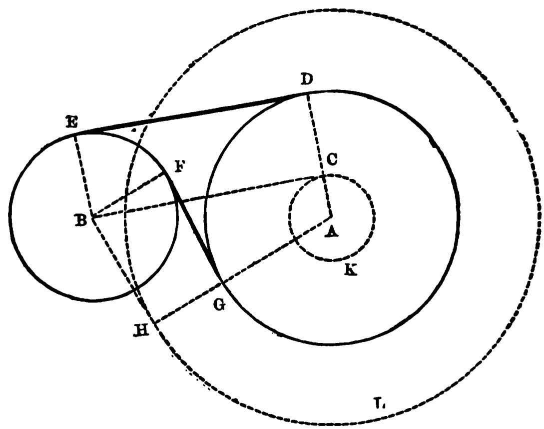
To draw a common tangent to two given circles.

Let A and B (Fig. 26) be the centres of the circles.

(1) Let the points of contact be on the same side of the line joining the centres; with centre A and radius A C equal to the

difference of the radii of A and B describe the circle C K; draw B C touching it in C; join A C and produce it to meet the circumference in D; draw B E parallel to A D, and join D E: D E will touch both circles, since E C is a rectangular parallelogram.

FIG. 26.



(2) Let the points of contact be on opposite sides of the line joining the centres; describe the circle L H with centre A and radius equal to the sum of the radii of A and B, and proceed as before.

EXAMPLES FOR PRACTICE.

1. Find, by geometrical construction, a fourth proportional to three straight lines, whose lengths are $2\frac{7}{8}$, $3\frac{1}{4}$, $1\frac{2}{3}$ inches, respectively (*Prob.* XXIX.).

2. Draw a line 3 inches long; assume a point about 2 inches above it, and from this point draw a perpendicular to it (*Prob.* V.).

3. Describe a circle 2 inches in diameter; then describe two circles, each equal to the first, touching it and each other.

4. Draw two circles having radii of 1 and 1·2 in. respectively, touching each other (*Prob.* VII.).

5. Draw 5 parallel lines at ·2 in. apart, the top line being thin and dotted, the bottom line thick and dotted, the middle one being thick, the intermediate lines thin.

6. Construct a triangle having its sides A B, A C, B C,=2, 3, and 3·5 inches respectively, and make an angle equal to the angle A B C (*Prob.* II.).

7. Describe a circle of 1·4 in radius, and draw tangents, intersecting each other, from two points in the circumference 90° apart (*Prob.* VIII.).

8. Erect a perpendicular to one of the tangents in *Ex.* 7, at a point ·5 in. distant from their intersection. Or, show how to let fall a perpendicular upon a straight line from a given point above it (*Prob.* V. 3).

9. Draw a straight line 2·5 in. long, and from its right extremity let fall a perpendicular=2·2 in (*Prob.* V. 4).

10. Draw a straight line, A B, 3 in. long, and from a point D, ·5 in. from A, draw a line D C, making an angle of 45° with D B, without using the protractor (*Prob.* V. 2; I.).

11. Assume two points 1·8 in. apart, and describe an arc of a circle with a radius of 2 in. passing through them.

12. Draw four parallel lines alternately dark and light. The lines to be 4 inches long.

13. Describe three circles touching one another, taking as their centres the angles of an equilateral triangle of 2 inches side. Ink in one circle in a dark line, the second in a light line, and the third in a dotted line (*Prob.* VII.).

14. Draw a line 4 inches long, and divide it, by construction, into 7 equal parts (*Prob.* VI.).

15. Draw 3 concentric circles, with radii equal to 1 in., $1\frac{1}{5}$ in., and $1\frac{2}{5}$ in. Ink in the outer circle with heavy dots, the middle circle with chain dots, and the inner circle with simple dots.

16. Draw a line 3 in. long, and at its middle point erect a perpendicular $1\frac{1}{2}$ in. high (*Prob.* V.)

17. Draw a circle 2 in. in diameter, and, by construction, draw a tangent to a point in its circumference (*Prob.* VIII.).

18. Draw a line 3 inches long intersecting a line 4 inches long at right angles (*Prob.* V.).

19. Draw two straight lines, A B, C D, each 6 inches long, inclined to each other in such a manner that A is 2 inches from C; and B 3 inches from D; and draw a straight line which, if prolonged, would pass through the intersection of their prolongation (*Prob.* XXI.).

20. Construct a triangle having no side less than $1\frac{1}{2}$ in., and no angle less than 30°; and describe a circle passing through its angular points (*Prob.* X.).

21. Draw 3 circles with radii equal to 1 inch, and each circle touching the other two.

22. Draw an irregular pentagonal figure having all its angles salient from the centre, and no side less than 1 inch long. Construct a square of the same area (*Prob.* XIX. and XVI.).

23. Describe a circle with a radius of 1·2 in.; assume a point $3\frac{3}{4}$ inches from its centre, and draw a tangent to it by construction (*Prob.* VIII.).

24. Find a mean proportional between two lines, $2\frac{1}{2}$ in. and 1 in. long; and figure its length on the diagram (*Prob.* XI.).

25. Construct a triangle having its 3 sides 2 in., $2\frac{1}{2}$ in., and 3 in., respectively, and describe a circle touching the sides.

26. Describe a circle of 1·5 in. radius. Draw two tangents, intersecting each other, from points 60° apart.

27. Draw arcs of 30°, 50°, 70°, with radii which are as 3, 2, 1; the middle arc to be of contrary flexure to the other two, and tangential to both of them, forming with them a continuous curve.

28. Construct a triangle of 4 sq. in. area, such that one of its angles is 40° and another 60°.

29. Draw a square of 3 in. side. Divide one of its diagonals into 7 equal parts, and draw straight lines through the points of division parallel to the other diagonal and terminated by the sides of the square.

30. Construct a triangle having two of its sides $3\frac{1}{2}$ in. and 3 in. respectively, and the angle opposite to the shorter of these sides equal to 50°. Figure the values of the third side and of the remaining angle.

31. Construct the polygon A B C D E F from the following data:—

Sides, A B=2 in., A F=1·7 in.
Diagonals, A D=4 in., A E=3·05 in.
Angles, A B C=$33\frac{1}{2}°$, B A D=$59\frac{1}{2}°$, B A E=$119\frac{1}{2}°$; B A F=125°.

Write down the magnitude of each side not given.

32. Construct a scale for the figure in *Ex.* 31, supposing A B to represent 300 yards.

33. Draw a line 4 in. long, and erect a perpendicular from its right extremity, without producing it.

34. Describe a circle of $1\frac{1}{8}$ in. radius touching each of the lines in *Ex.* 33.

35. Construct a triangle of which the sides shall be $3\frac{1}{2}$, 3, and $2\frac{1}{2}$ in. respectively. Ascertain, with the protractor, and note on the triangle the values of the two greater angles.

36. Describe a circle with a radius of $1\frac{1}{8}$ in.; draw a tangent to it, by construction, from a point 3 inches from the centre.

37. Describe a circle of 2 in. radius; divide the circumference into 12 equal parts; draw radii through the points of division, but terminate them at the circumference of a concentric circle of $\frac{1}{8}$ in. radius.

38. Draw a triangle having one angle=55°, and the sides containing this angle $1\frac{3}{4}$ in. and 2 in., respectively. Find the centres of the inscribed and circumscribed circles.

39. A line 200 ft. long is represented on a drawing by a line 5 in. long. Make a scale of feet for the drawing and give its representative fraction.

40. From a point A draw two straight lines A B, A C, each equal to 2·5 in., and making with each other an angle B A C=45°. Bisect the angle without using the protractor (*Prob.* I.).

41. Describe a circle 3 in. in diameter, assume a point in its circumference, and through this point draw a tangent to it (*Prob.* VIII.).

42. Draw a line 3·75 in. long, and divide it, by the method of describing arcs, into 4 equal parts (*Prob.* IV.).

43. Construct a five-sided rectilineal figure having its sides equal to 2, $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, and 3 in., respectively, and two of its angles right angles. Reduce this figure to a triangle (*Prob.* XIX.).

44. With a radius of 6 in. describe an arc of a circle of about 80°, and draw a straight line tangential to the arc at one of its extremities (*Prob.* VIII.).

45. Using the arc described in *Ex.* 44, find by construction the centre of the circle of which it is the arc (*Euc.* III. 1, *Cor.*).

46. Construct angles of 15°, 30°, 45°, 60°, 120°, without the aid of the protractor.

47. Construct a square on a side of 1·2 in., and describe a circle about it (*Prob.* XXIII.).

48. Draw, on a scale of 40 yards to an inch, a triangle of which each side equals 120 yds. Round off one of the angles with the arc of a circle touching the two sides forming that angle. Ink in the figure with a dark line (*Prob.* XIII.).

49. Draw, on the same scale, 5 figures similar to the above and external to it, at distances from it of 5, 7, 9, 10, 15 yds. respectively. Ink in the first two of these five figures in dotted lines; those at 9 and 10 yds. distant in fine lines, and the exterior one in a dark line.

50. Draw a circle having a diameter of $2\frac{1}{4}$ in., and supposing its centre unknown, show how to find it (*Euc.* III., 1, *Cor.*).

51. Draw a line 3·75 in. long, and divide it into 9 equal parts (*Prob.* VI.).

52. Draw a line A B, $2\frac{1}{2}$ in. long, assume a point C, $\frac{1}{2}$ in. above B, in such a position that a line joining B and C would be perpendicular, or nearly so, to A B. Describe a circle passing through C and touching A B, at a point $\frac{3}{4}$ in. from A (*Prob.* VIII.).

53. Draw on a scale of 45 yds. to an inch an irregular figure, A B C D E F, according to the following instructions:—A B = 145 yds., B C = 117 yds., the angle A B C = 135°, C D = 88 yds., the distance A D = 202 yds, D E = 141 yds., the distance A E = 148 yds., E F = 84 yds., F A = 87 yds.

54. Reduce the figure in *Ex.* 53 to a triangle: figure the sides of the triangle and its area (*Prob.* XIX.).

55. Draw a straight line 3 in. long, and from a point nearly opposite to, and 2 in. from, one of its extremities, let fall a perpendicular to it (*Prob.* V. 3).

56. Assume 3 points A, B, C, such that A is $2\frac{1}{4}$ in. from C and

$1\frac{1}{2}$ in. from B, B being also $1\frac{3}{4}$ in. from C. Describe a circle passing through these points (*Prob.* X.).

57. Draw in pencil the figure denoted by the following measurements, A B=250 yds., B C=150 yds., A C=300 yds., C D =145 yds., the angle B C D=120°; D E=225 yds., B E=310 yds. Find the distance from A to E and note it in yds. Scale 100 yds. to an inch.

58. Ink in the lines A B, B C, C D, D E, and join E A. Reduce the figure A B C D E to a triangle of equal area, and note its contents in square yards (*Prob.* XIX.).

59. Draw two lines inclined to each other at an angle of 30°, but not prolonged to their intersection. Join them by the segment of a circle, tangential to them, with a radius of 1 inch.

60. Construct, as accurately as you can, a triangle of which the lengths of the sides are $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$ in., respectively. The figure to be inked in with the finest lines you can draw.

61. Inscribe a circle in the above triangle, inking in the circle with a thick line. (*Bisect any two of the angles.*)

62. Describe a circle with a radius of $1\frac{1}{4}$ in., and draw a chord cutting off from it an arc of 38° (*Euc.* III. 34).

63. Draw a straight line 4 inches long, and divide it in the proportion of the numbers, 3, 5, 2, 8. Figure the parts (*Prob.* XV.).

64. Through any 3 points A, B, C, not in the same straight line, describe a circle; and explain how this problem may be applied to the case of a circular arch of given span and rise.

65. Draw a square of 2 in. side, and within it 4 equal circles, each touching two others and two sides of the square.

66. Divide a line A B, 3·6 in. long, into 5 equal parts by construction.

67. In a circle of $1\frac{1}{4}$ in. radius draw a chord so that the angle which it subtends at the circumference may be 50°.

68. On a line $1\frac{1}{2}$ in. long draw three isosceles triangles having their vertical angles 19°, 25°, and 40°, respectively. Ascertain and figure the distance from the apex of each triangle to the common base.

69. Draw two tangents to a circle of 1·75 in. radius, the tangents to contain an angle of 50°.

70. Take two points 2·5 in. apart, and describe two circles with radii ·75 in. and 1·25 in., respectively, with these points as centres. Draw a circle of 1·3 in. radius to touch both of these circles. Ink in the first two circles with a thin line and the last with a thick one.

71. Divide a line 3·85 in. long into 6 equal parts. Supposing a line so divided to be a scale of yards, how many yards to an inch would the scale represent? Give its representative fraction.

72. Divide a line 4·3 in. long into 7 equal parts, by construction.

73. In a circle of 1·7 in. radius, inscribe three equal circles, each touching the circumscribing circle and the two inscribed circles.

74. Draw two tangents to a circle of 2·5 in. radius, so that they may contain an angle of 45°.

75. Find a mean proportional to two lines 2 in. and $1\frac{1}{4}$ in. long, respectively.

76. State, in the form of a fraction, the ratio of the lines on a drawing to the lines of the object represented, supposing the drawing to be made on a scale of 33·33 yards to an inch.

77. Draw a triangle having an angle of 50°, and the sides containing it 2 in. and 2·5 in. long. On the greatest side of the triangle construct a parallelogram equal in area to the triangle.

78. Draw two parallel lines $1\frac{3}{4}$ in. apart, by a rigid construction, without using parallel rulers; the lines to be about 3 in. long, and inked, one with a thin line, the other with a thick one. Dot a circle touching both lines, and show the construction for finding its centre.

79. Describe a triangle having two sides $2\frac{1}{2}$ in. and $3\frac{3}{4}$ in. long, respectively, the angle subtended by the longer side being 85°.

80. The sides of the rhomboid being $2\frac{3}{4}$ in. and $3\frac{1}{2}$ in., respectively, and one of its angles being 70°, find the lengths of the diagonals by measurement. What are the values of the other angles?

81. Describe a triangle of which the sides are 2·75, 3 and 1·98 inches, respectively, and bisect it by a line parallel to the shortest side.

82. Construct a regular heptagon on a line 1·75 in. long, and another of half its area.

83. Draw a square of 5·79 inches area, and a second of $\frac{3}{7}$ its area.

84. Two adjacent sides of a trapezium are 2·3 and 3·04 inches, respectively, and include an angle of 60°, the other sides are 3 and 3·5 inches. Describe this figure and bisect it by a line drawn from its greatest angle.

85. The vertical angle of a triangle is $\frac{2}{3}$ of a right-angle, its sides are 2·89 and 3·47 inches, respectively; find the length of the base, and through the middle point of the side 3·47 in. draw a line cutting off $\frac{4}{11}$ of the triangle.

86. Two angles of a triangle are 50° and 70°, the adjacent side 2·37 inches; draw a line making an angle of 80° with this side and bisecting the triangle.

87. An isosceles triangle has a base of 3 inches and a vertical angle of 57°; describe it and an equivalent equilateral triangle.

88. One triangle has a base of 2·4 in., a vertical angle of 35°, its sides in the ratio of 4 : 7. Another has a base of 3·33 inches, one of the adjacent angles 75°; describe these two triangles, and a third similar to the first and equal to the second.

89. Divide a triangle whose sides are 4, 3·75, and 2·99 inches into 5 equal parts by lines parallel to the longest side.

90. Describe a square whose area is 7·69 inches, and place a regular octagon in it; find the area of the octagon.

91. Describe a regular pentagon equal to a triangle whose sides are 4·05, 3·98, and 2·75 inches, respectively.

92. Describe a circle of 1 inch radius touching two circles of 2·5 and 2·49 inches diameter, respectively. Can this always be done?

93. A and B are two points 3 inches apart; A being distant 2·5 inches from the centre of a circle of 1·04 inches radius. Describe a circle passing through A and B and touching this circle.

94. Find the length of the side of the square inscribed in the quadrant of a circle of 2 in. radius.

95. Inscribe a square in a segment of a circle whose base is 2·25 inches and in which the angle is 40°.

96. Inscribe a circle in the quadrant of a circle whose radius is 2·5 inches.

97. Inscribe a circle in the sector of the same circle in which the angle at the centre is 70°.

98. From a point 3 inches from the centre of a circle whose radius is 2·3 in. draw a straight line which shall be cut in extreme and mean ratio by the circle.

99. Describe a circle of 1·25 inches radius and another with the same centre, such that the ring between them shall be one-fifth of the area of the first circle.

100. The centres of two circles, whose radii are 1·75 and 1·35 inches, are 3·1 inches apart, draw a common tangent to them.

Questions set at Woolwich Entrance Examinations from 1866 *to* 1873.

1. Draw a line 3 in. long and bisect it.

2. Draw a triangle, the sides of which are 2·5, 3, and 3·5 in., and construct another triangle of equal area, having one of its sides 3·75 in. long.

3. Describe a circle of $1\frac{1}{2}$ in. radius. Draw a straight line $1\frac{1}{2}$ in. long within the circle, and terminating at both ends in the circumference.

Erect a perpendicular from the middle point of this line.

4. Describe a circle with a radius of 1 in., and about it describe a triangle equiangular to the first triangle in *Ex.* 2.

5. Describe an isosceles triangle having its base 2 in., and each of the other sides 3 in. long. Describe also an isosceles triangle having the angle at its vertex double that at the vertex of the triangle first described.

6. Describe a circle with a radius of $1\frac{1}{2}$ in., and then describe an equilateral and equiangular pentagon about it.

7. Describe a segment of a circle which shall contain an angle equal to one of the angles of an equilateral triangle.

8. Draw a straight line 2·75 in. long, and divide it into two parts, such that the rectangle contained by the whole and one part shall be equal to the square on the other part.

9. Draw a straight line parallel and equal to any straight line you choose to consider as given, and 1 inch from it.

10. Draw an isosceles triangle having the angles at the base each 70°, and the two equal sides each $2\frac{1}{2}$ in. long. Measure and figure the length of the base.

11. Describe a circle with a radius of $1\frac{1}{2}$ in., and from it cut off a segment which shall contain an angle of 35°. Find by measurement, and figure the number of degrees in the arc of the segment cut off.

12. Draw a line $3\frac{1}{2}$ in. long, and from it cut off $\frac{1}{5}$ of its length.

13. Find a third proportional to two lines $1\frac{1}{2}$ in. and 1 in. long, respectively. Figure the length of the third proportional from measurement.

14. Draw a rectilineal figure A B L H G from the following measurement and angles:—

Side A B=2 in.; angles, B A G=83°,
A B G=25°, G B H=30°, B G H=83°,
H B L=29°, B H L=42°.

Figure the value of the angle at L from measurement, and also the lengths of the sides B L, H L.

15. Divide a straight line A B, $2\frac{1}{3}$ in. long, into two parts, in the point C, so that the rectangle A B, B C may be equal to the square on A C. Find, by measurement, the length of the diagonal of the square on A C.

16. Inscribe a circle in a square of $2\frac{3}{4}$ in. side.

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17. Describe an equilateral triangle of $2\frac{1}{4}$ in. side.

18. Draw a line C, about 1 in. long, and a line A B about $1\frac{3}{4}$ in. long; cut off from A B a part equal to C.

19. Draw a line 2·75 in. long and bisect it.

20. Make a triangle of which the sides are as 5, 6, 7, and measure the values of its angles with a protractor. (*The values of the angles to be neatly entered on the diagram.*)

21. Make an angle equal to the least of the angles in *Ex.* 20, at a point A in an assumed line A B, 3 in. long.

22. Describe an equilateral and equiangular pentagon of $1\frac{3}{4}$ in. side, and inscribe a circle in it.

23. Describe a circle with a radius of $1\frac{1}{3}$ in., and in it inscribe an equilateral and equiangular quindecagon.

24. Divide a straight line 5 in. long into 3 parts in the proportion of 3, 2, and 5.

25. On a straight line $2\frac{3}{8}$ in. long, describe a segment of a circle which shall contain an angle of 65°.

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26. Draw a line A B, $3\frac{3}{4}$ in. long, and through a point C, 1 in. from it, draw another line 3 in. long parallel to it.

27. Construct a triangle of which the sides are as 4, 5, 6, their sum being 8 in.; to a straight line 1 in. long, apply a parallelogram equal to this triangle, and having one of its angles equal to 75°.

28. Construct a rectilineal figure A B C D, having the sides A B, A D, each 2 in. long, and B C and D C, each $2\frac{1}{2}$ in., the angle B A D being equal to 70°. Describe a square equal to the figure A B C D.

29. Describe a circle of $1\frac{1}{4}$ in. radius, and from a point 4 in. from its centre draw a straight line touching the circle.

30. Describe a circle of $1\frac{1}{2}$ in. radius, and construct a triangle with its apex on the circumference, and with its base, 2 in. long, subtending a portion of the circumference. Find the value of the arc so subtended, with the protractor, and write it down in a neat print-hand.

31. Draw a triangle having an angle equal to 58°, and the sides containing this angle $2\frac{1}{2}$ in. and $3\frac{1}{4}$ in. long, respectively.

32. Draw an equilateral and equiangular pentagon of $1\frac{1}{4}$ in. side, and inscribe a circle in it.

33. Construct a polygon A B C D E F from the following conditions:—

Sides, A B=2 in., A F=1·75 in.

Diagonals, A C=3 in., A D=3·75 in., A E=3 in.

Angles, B A C=35·5°, B A D=59·5°, B A E=99·5°, B A F=125°.

Write down, in neat print-hand, the magnitudes of the sides not given, and those of the angles A F E and F E D.

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34. Describe an equilateral triangle on a straight line $2\frac{1}{2}$ in. long.

35. Assume any convenient point on your paper, and from it draw a straight line equal to the side of the triangle. *Ex.* 34.

36. Draw a rhomboid of which the longer sides are 3 in. in length and the shorter 2 in., and the two acute angles each equal to 50°.

Ascertain and write down the length of the longer diagonal.

37. Describe a triangle similar to that required by *Ex.* 34, and equal to the rhomboid required in *Ex.* 36.

38. Find a third proportional to two lines $1\frac{1}{2}$ and $1\frac{3}{4}$ in. long, respectively.

39. Describe an isosceles triangle of which the angles at the base are each double of the third angle, and the two equal sides each 2 in. long.

40. Describe a circle of $1\frac{1}{2}$ in. radius, and in it inscribe an equilateral and equiangular pentagon.

41. Describe a circle of $1\frac{1}{2}$ in. radius, and from a point 2 in distant from its centre draw a tangent to its circumference.

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42. Draw a line A B, $3\frac{1}{2}$ inches long; assume a point C about $1\frac{1}{2}$ inch from it, and through C draw C D parallel to A B. Draw also six other lines between A B and C D parallel to these lines, and at $\frac{1}{10}$ inch apart.

43. Set off with your protractor an angle of 57°, and bisect this angle. Again bisect the two angles thus obtained.

44. Draw a square of 3·3 inches side. In it describe four equal circles each touching one side of the square and two of the other three circles. Within each of these four circles and with their centres, but with a shorter radius by $\frac{1}{8}$ inch, describe a second circle. Ink in the larger circles with a thin line and the smaller ones with a thick line.

45. Construct a rhombus having an angle of 65° and a base of 3 inches. Measure its two diagonals accurately, and write down their lengths.

46. Draw a triangle having one of its angles=52°, and the two sides containing this angle 1·8 and 2·6 inches. Find the centres of the inscribed and circumscribing circles.

47. Assume three lines, 2 inches, 1 inch, and 2½ inches long respectively, and find a fourth proportional to them.

48. Divide a line 5 inches long into seven equal parts, and describe seven equal circles having their centres in this line, and cutting it at the points of division.

49. Construct a right line figure from the following conditions:—

The side A B=2 inches.	The angle A B C=120°.
„ B C=1·75 „	„ B C D= 90°.
„ C D=2·2 „	„ C D E=120°.
„ D E=1·2 „	

Write down the length of the remaining side E A, and the values of the angles D E A, E A B.

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50. Draw two parallel lines at 1½ inch apart, and between them insert nine other lines parallel to them, and dividing the space between them at equal intervals. Ink in the nine lines with thick, thin, and dotted lines alternately.

51. Draw three circles, each with a diameter of 2 inches, so that each circle may touch the other two.

52. Draw a square of 2¾ inches side, and in it place four equal circles, each one touching two of the other circles and two sides of the square, and also four equal circles, each one touching two other circles and one side of the square.

53. Describe a circle with a radius of 1½ inch, and place in it five parallel chords, so that the angles subtended by them at the circumference of the circle may be equal to 20, 35, 50, 65, and 80 degrees respectively. Measure and write down the lengths of the chords.

54. Construct a triangle of which the sides are 2·7, 2·4, and

3·5 inches respectively. Measure with your protractor and write down the value of its three angles.

55. Draw a parallelogram having one angle 60°, and the sides 2 and 3 inches long. Measure and write down the lengths of the diagonals. Make a square equal to the parallelogram.

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56. Construct an isosceles triangle of which the base is 2 inches, and each of the sides 2·8 inches. Measure and write down the value of each of the angles.

57. Divide a straight line 3·6 inches long into 7 equal parts, and through each point of division draw a line $2\frac{1}{4}$ inches long, making an angle of 50° with the first line, these parallels being alternately thick, thin, and dotted lines.

58. Draw the segment of a circle with a radius of 2 inches to contain an angle of 100°. Inscribe a circle in the figure formed by the arc of the segment and the radii at its extremities.

59. From a point A 3 inches above the extremity B of a straight line B C 1·9 inches long draw a perpendicular to B C, without producing it. Bisect A B in D, describe a circle passing through D, B, C, and measure the length of its radius.

60. Construct the rectilineal figure A B C D from the following conditions :—

A B=2·1 inches.	The angle A B C=125°
A C=2·8 „	„ A C D= 52°.
C D=1·7 „	

Write down the lengths of B C, A D, and the size of the angles B A D, A D C.

61. Construct a triangle with a base of $1\frac{1}{2}$ inch, having each of the angles at the base double of that at the vertex. Find a mean proportional between the base and one of the sides, and measure its length.

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62. Bisect a straight line A B, 3 inches long, in C; through C draw C D, 2·4 inches long, making the angle D C B, 65°. Join A D and D B, measure and write down their lengths and the sizes of the angles of the triangle A D B.

63. Divide a straight line $3\frac{1}{3}$ inches long into 8 equal parts, and through the extremities of the line and through the points of division draw parallel straight lines, alternately continuous, dotted and chain dotted, $2\frac{5}{8}$ inches long, and making angles of 40° with the first line.

64. Construct a triangle the sides of which measure 2, 2·6, and 3·1 inches, and to a straight line 1·7 inches long, apply a parallelogram of equal area, having one of its angles 58°.

65. Draw the segment of a circle to contain an angle of 46°, the chord measuring 1·7 inches. Measure and write down the length of the radius of the circle.

66. To a circle of 1·8 inches radius apply an inscribed pentagon and circumscribed hexagon.

67. Draw a square of 3·8 square inches area, and an equilateral triangle equal to it.

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68. Divide a straight line 5·3 inches long into five equal parts, and upon each of these parts as a diameter describe a circle.

69. Upon a straight line $1\frac{7}{8}$ inches long construct a square; divide it by parallel straight lines alternately thin and dotted, into seven equal rectangles.

70. Construct the triangle A B C with sides A B, B C, two and three inches each, and the included angle A B C 125°. Trisect the base A C in the points D and E, join B D and B E; measure and write down the sizes of the angles A B D, D B E, and E B C, and the lengths of D B and E B.

71. Describe a circle with a radius of 1·6 inches. Inscribe in it a regular hexagon, and circumscribe around it a regular octagon. Write down the value of the angle of each polygon.

72. Find a third, and a mean, proportional to two straight lines 1 inch and $1\frac{1}{2}$ inch long. Measure and write down their lengths.

73. Construct an equilateral triangle equal in area to the square in Question 69. Show clearly the steps of the construction.

74. From the circle in Question 71, cut off a segment to contain an angle of 72°.

CHAPTER II.

ON THE USE OF THE SECTOR, THE PROTRACTOR, AND THE MARQUOIS SCALES.

Definition. The SECTOR is an instrument formed of two flat rulers or legs, of equal length, fixed to a common centre and movable about that centre in a plane.

Def. SECTORAL LINES are lines drawn in pairs from the centre, one of each pair on each leg. The most important of these are the line of lines, marked L; the line of chords marked C; and the line of polygons, marked Pol.

Instead of a single line, for a sectoral line on each leg, three parallel lines are drawn, to show clearly the divisions of the line. In all cases the points of the dividers must be applied to the innermost of these, that being the one which is drawn from the centre.

Def. A LATERAL DISTANCE is a distance measured from the centre along any sectoral line.

Def. A TRANSVERSE DISTANCE is a distance measured from a point in one line of a pair to the corresponding point in the other line.

Explanation of the principle of the LINE OF LINES.

Let P (Pl. II. Fig. 2) be the centre, P L, P L′, the line of lines, divided into ten equal parts in the points 1, 2, 3, 4, 5, 6, 7, 8, 9 (in the sector constructed for use each of these primary divisions is divided into ten equal secondary divisions, so that the line of lines is divided into 100 equal parts):

take $P\,l' = P\,l$ and $\mathrm{P\,L'} = \mathrm{P\,L}$; draw $\mathrm{L\,L'}$, $l\,l'$;

then, $\mathrm{L\,L'}$ is parallel to $l\,l'$, and consequently

$$l\,l' : \mathrm{L\,L'} :: P\,l' : \mathrm{P\,L'};$$

therefore, whatever part $p\,l'$ is of P L′, $l\,l'$ is the same part of L L′.

APPLICATION OF THIS PRINCIPLE.

1. Let it be required to bisect a given straight line.

Open the sector until the transverse distance at 10 is equal to the given line; then the transverse distance at 5 will be equal to one half of the line.

N.B. The transverse distances at 8 and 4, 6 and 3, or 4 and 2, may be employed for the bisection.

2. To divide a straight line into any number of equal parts.

Let the required number of parts be 9.

Make the transverse distance at 9 equal to the given line; then the transverse distance at 1 will be equal to $\frac{1}{9}$ of the line.

This construction may be effected more accurately by making the transverse distance at 9 equal to the given line, as before, and then setting off from each end of the line the transverse distance at 4. The line will thus be divided into three parts, the middle one of which will be $\frac{1}{9}$ of the line, each of the others $\frac{4}{9}$ of it.

Cor. This shows how from a given line to cut off any aliquot part.

3. To find any fraction of a given line.

Ex. 1. $\frac{3}{5}$ of a line 4·25 inches long.

Make the transverse distance at 5 equal to the line, the transverse distance at 3 will be $\frac{3}{5}$ of the line.

Ex. 2. To find $\frac{9}{23}$ of a line 5·17 inches long.

Since there are only ten primary divisions, recourse must be had to the secondary divisions, to solve this problem. In order to bring the construction some distance from the centre, which will increase the accuracy of the result, multiply the numerator and the denominator of the fraction by some number which will not make the denominator when so multiplied greater than 100;

in this case 4 will be a convenient multiplier; then $\frac{9}{23}=\frac{36}{92}$; make the transverse distance at the secondary division 92 equal to 5·17 inches, the transverse distance at the 36th secondary division will be $\frac{36}{92}$, that is $\frac{9}{23}$, of 5·17 inches, as required.

4. To find a fourth proportional to three given straight lines.

Let A, B, C be the lines, make the transverse distance at the lateral distance A equal to B, then the transverse distance at the lateral distance C will be the fourth proportional required.

Cor. If a third proportional to A and B be required, the solution will be performed in a similar way, for in this case B = C, and therefore the transverse distance at the lateral distance B must be taken for the third proportional.

N.B. In a correctly divided sector the line of lines will be found a convenient instrument for solving Problems IV., VI., XXVII., and XXIX., of Chap. I.

THE LINE OF CHORDS.

The line of chords is chiefly used for setting off angles of a given number of degrees. It is so constructed that if the sector be opened until the transverse distance at 60 is equal to the chord of 60° in any circle, the transverse distance at any other number on the line of chords will be equal to the chord of that number of degrees in the same circle. For example, the transverse distance marked 20 will be the chord of 20°.

Ex. 1. To make an angle of 35° (Pl. III. Fig. 1).

With the point M, in the straight line M N as a centre, and any radius M P, less than M N, describe a circle P T.

Make the transverse distance at 60 equal to M P, because the chord of 60° is equal to the radius.

With the hair dividers make P Q equal to the transverse distance at 35, join M Q.

The angle N M Q will be the angle required.

Ex. 2. To make an angle greater than 60° but less than 90°.

Let the angle be 75°.

From M (Pl. III. Fig. 2) draw M Q at right angles to M N.

By the preceding example make the angle Q M R equal to 15°; the angle R M N is the angle required; for $90° - 15° = 75°$.

Ex. 3. To make an angle greater than 90°.

Let the angle be 133°.

Draw M Q (Pl. III. Fig. 3) at right angles to M N; make the angle Q M S equal to 43°.

The angle N M S is the angle required; for $90° + 43° = 133°$.

Examples 2 and 3 might have been solved by making an angle equal to $\frac{1}{2}$ or $\frac{1}{3}$ of the required angle, and setting this off along the arc as many times as necessary. But since this method sometimes gives rise to fractions of a degree, it will generally be found more convenient to adopt the constructions given.

Cor. It is evident that an angle of a given number of degrees may be readily divided into 2, 3, 4, 5, &c. equal parts by means of the line of chords.

THE LINE OF POLYGONS.

This line is chiefly used to divide the circumferences of circles into equal parts, for the purpose of describing regular polygons. It is constructed in such a manner that if the sector be opened until the transverse distance at 6 is equal to the radius of any circle, that is, the side of a regular hexagon (*Euc.* IV. 14) inscribed in that circle, the transverse distances at 5, 7, 8, 9, 10, 11, 12 will be respectively equal to the sides of a regular pentagon, heptagon, octagon, &c. &c., inscribed in the same circle.

Ex. 1. To inscribe a heptagon in a given circle.

Let S (Pl. III. Fig. 4) be the centre of the circle A D F; S A its radius.

Open the sector until the transverse distance at 6 is equal to S A; then the transverse distance at 7 will be equal to the side of the heptagon.

Let A B be this distance: place around in the circle straight lines B C, C D, D E, E F, F G, G A, each equal to B A.

The figure A B C D E F G will be the heptagon required.

Ex. 2. To describe a regular nonagon upon a given straight line.

Let A B (Pl. III. Fig. 5) be the given line.

Open the sector until the transverse distance at 9 is equal to A B. With A and B as centres, and a radius equal to the transverse distance at 6, describe two circles intersecting in S.

With S as a centre and the same radius as before, describe the circle A D G K. Place around in it the straight lines B C, C D, D E, E F, &c. &c., each equal to A B: A B C D E F G H K will be a regular nonagon.

Obs. If A be the number of degrees in an angle of a regular polygon of n sides, $A^\circ = 180^\circ - \frac{360^\circ}{n}$ (*Euc.* I. 32, *Cor.* 1), consequently when $\frac{360}{n}$ is an integer, a regular polygon may be described upon a given straight line by the following method:—

Ex. Let $n=8$, therefore, $A^\circ = 180^\circ - 45^\circ = 135^\circ$;
let A B (Pl. III., Fig. 6) be the given line.

Make the angle A B P equal to 135°. Take B C equal to A B; describe the circle A C E G H, passing through the points A B C; place around in this circle the lines C D, D E, E F, F G, G H, H A, each equal to A B or B C. The figure A B C D E F G H will be a regular octagon.

THE MARQUOIS SCALES.

A set of these consists of two rectangular rulers and a right-angled triangle. Each ruler has two natural and two artificial scales engraved on each side of it, with figures in the middle of the former to denote the number of parts into which the inch is divided. The scales given are those of 30, 60, 25, 50 parts to an inch on one ruler, and 20, 40, 35, 45 on the other: from which can also be obtained those of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, and 15, making in all twenty scales. The natural scales are divided in the ordinary manner, the left-hand primary division being subdivided into ten parts; but the artificial scale has all the primary divisions, each of which is equal to three of those of

the natural scale adjoining, subdivided into tenths, each of these is therefore equal to three of the natural subdivisions; the zero point is in the middle of the scale. The divisions are numbered both ways. The longest side of the triangle is three times the shortest side, and has a short line drawn perpendicular to it at its middle point.

Let A B (Pl. II. Fig. 3) be the edge of the ruler, P Q R the first position of the triangle, *p q r* the position into which it has been moved.

Draw *p s* parallel to Q R, then by similar triangles—

$$P p : p s :: P R : R Q;$$

but P R is three times R Q, therefore P *p* is three times *p s*. In order, therefore, to draw parallel lines at given distances apart, place the star line on the triangle, against one of the divisions on the artificial scale, and, holding the ruler firmly in its place, slip the triangle along as many divisions as the given distance occupies in the natural scale. It will be found that its bevelled edge has moved in a direction perpendicular to itself, through the required distance, or one-third of that travelled by the star line along the ruler.

THE PROTRACTOR.

This instrument is used for the purpose of protracting or laying down angles of a given number of degrees by the method of coincidence. The construction of it is indicated in Pl. II. Fig. 1.

Ex. Let it be required to make an angle of 40° at the point P, in the line M N; with the centre on P, move the protractor until the line radiating from the centre, and marked 40°, coincides with M N; draw P Q along the edge, the angle Q P M will be the angle required.

If the given line be P Q, and not long enough to admit of applying the instrument in this manner, place the centre on P, as before, then with a very finely pointed pencil, or with the hair dividers, mark a point on the paper, where the radiating line through 40° meets the edge. Remove the protractor, and draw the line M N through P and the point thus found. M P Q will be the angle.

EXAMPLES FOR PRACTICE.

1. Form a triangle whose sides are 2·53 and 1·78 in., and the included angle 132°; describe a circle about it (*Prob.* X.).

2. Draw a figure, A B C D E, of which the side A B= 1·4 in., B C=1·5 in., C D=1·4 in., D E=1 in., and E A= 1·5 in.; the sides A B and A E to contain a right angle, A E and D E an angle of 130°.

3. Draw a line 3 in. long, and at its left-hand extremity make an angle = the angle C D E of the last figure. Reduce the figure to a triangle (*Prob.* II. and XIX.).

4. Draw 3 concentric circles, whose radii are $\frac{3}{4}$ in., 1 in., and $1\frac{1}{4}$ in. respectively. In each circle inscribe a hexagon, the sides of the 3 hexagons being parallel. Draw radii to each angle of the outer hexagon. The inner circle and the radii are to be dotted.

5. Construct an isosceles triangle on a base of 2 in., the angle at the vertex being 100°. Make a rectangle which shall contain the same area as this triangle (*Prob.* XXII. and XVI.).

6. Construct a triangle having two of its sides equal to 3 in. and 2·75 in., respectively, and the included angle=50°. Figure the remaining angles and the length of the third side.

7. Draw two lines each 3 in. long, forming an angle of 25°. Describe a circle of $\frac{1}{2}$ in. radius touching both lines (*Prob.* XIII.).

8. Construct a triangle having two of its angles equal to 38° and 100°, and the side opposite the greater angle equal to 3 in. Find the lengths of the other sides and figure them.

9. Standing at a point A on the bank of a river, I set off a line in a direction at right angles to the line joining A and the point B, on the opposite bank. On the line thus set off I measure a distance of 40 yards to C, at which point the angle between A and B is ascertained to be 43° 30′. Find, by construction, the distance across the river from A to B. Scale 20 yds. to an inch.

10. Construct a triangle having two of its angles equal to 43° and 78°, respectively, and the side opposite the larger angle= 4 in. Find the lengths of the other sides and figure them. The triangle to be inked in with the thinnest lines you can draw.

11. Draw a parallelogram of which one side shall measure $2\frac{1}{2}$ in., and the distance between it and the opposite side shall be $1\frac{1}{2}$ in. One angle of the figure to be 130°.

12. Draw a straight line A B, 2·5 in. long, and a second line B C, of the same length, making an angle of 18° at B. Describe a circle with a radius of 1·5 in., touching the line B C at a point D, 1·6 in. from B; and draw a second circle which shall also pass through D and touch A B (*Prob.* VIII.).

13. Two straight lines intersect at an angle of 35°; draw a circle of 2·25 in. radius touching both lines (*Prob.* XIII.).

14. Construct a regular heptagon on a side of 2 in., and explain the mode of construction.

15. Construct a triangle two of whose sides are 2·75 and 3·2 in., respectively, and the included angle 65°. Describe a circle about the triangle showing how the centre is found (*Prob.* X.).

16. Inscribe a nonagon in a circle of 2 in. radius.

17. Explain the principle of the '*line of lines*' on the sector, and from a line 4·37 in. long cut off portions respectively equal to $\frac{3}{17}$ and $\frac{4}{19}$ of its length.

18. Draw an arc of 73° with a radius of 3·36 in., and one of 100° touching the former at one extremity, radius 2·5 inches.

19. By means of the line of chords on the sector draw an angle of 102°, and an arc of 2·25 in. radius touching both lines of the angle. Mark the points of contact of the arc and the straight lines (*Prob.* XIII.).

20. Divide a line 4·13 in. long, so that the parts may be as 3, 7, 2, 17, $4\frac{1}{2}$ (*Prob.* XV.)

21. From a circle of 1·5 in. radius cut off two segments containing angles of 32° and 94°, respectively (*Euc.* III. 34).

22. Make an angle of 75°, bisect it with the compasses and ruler alone, and describe a circle of 1 in. radius tangential to both lines containing the angle (*Prob.* I. and XIII.).

23. Describe a segment of a circle having a base of 2·36 in., and containing an angle of 115° (*Prob.* IX.).

24. Construct a regular octagon, with a side 1·78 in.; reduce the figure to an equivalent triangle (*Prob.* XIX.).

25. Draw a straight line 4 in. long, and at one extremity of it set off an angle of 33°, using the line of chords on the sector, or the protracting scale.

26. Construct a nonagon on a right line 1·4 in. long by means of the line of polygons on the sector.

27. Construct an isosceles triangle on a base=$2\frac{1}{2}$ in., the angle at the vertex being 81° 30′. One of the angles to be set off with a scale of chords, the arcs used for the purpose being shown in dotted lines (*Prob*. XXII.).

28. Construct a regular pentagon on a base of $1\frac{1}{2}$ in., and describe a circle about it.

29. Construct an octagon, with a scale of chords, on a line ·9 in. long. Reduce it to an equivalent square (*Prob*. XIX., XVI., XI.).

30. Construct a triangle having two of its angles 35° 10′ and 114° 20′, and the side opposite the larger angle=2·9 in.; find the lengths of the other sides with your dividers and ivory scale.

31. Draw two lines each 6 in. long and meeting at an angle of 20°. Describe a series of circles touching these lines and each other, commencing with a radius of $\frac{1}{8}$ in. (*Prob*. XIII.).

32. Construct a regular hexagon of which the side is equal to 13 feet; and a triangle of equal area. Scale 10 feet to an inch. (*Prob*. XIX.).

CHAPTER III.

SCALES.

When the dimensions of an object, to be represented on paper, are so great that it would be either inconvenient or impossible to make a full-size drawing of it, the usual practice is to construct a drawing, each line in which has a given ratio to the line which it represents. In order that such a drawing may be generally intelligible, draughtsmen employ two methods, by either of which the absolute length of any line in the original may at once be determined from the draught.

The first method is to attach to the drawing a fraction, called the *Representative Fraction*, which expresses the ratio of any line in the drawing to the corresponding line in the original. Thus the fraction $\frac{1}{24}$ attached to a plan would show that the distance between any two points in the drawing was $\frac{1}{24}$ of the distance between the points in the original; so that 1 inch would represent 24 inches; 1 foot, 24 feet, &c.

By the second method, in addition to the representative fraction, a graduated straight line, termed a *Scale*, is annexed to the drawing for the purpose of conveniently measuring distances. The length of the unit in this scale must evidently bear the same ratio to the real unit of length that a line in the drawing bears to the line which it represents.

Scales are usually, though not necessarily, constructed of such a length as to represent a distance which is a multiple of ten lineal units of some kind; as 80 miles, 50 yards, 100 toises, 500 versts.

The construction of these scales, called Plain Scales, will be best illustrated by examples, of which several are subjoined.

EXAMPLES.

1. To construct a scale of $\frac{1}{24}$, or 2 feet to an inch.

The number of feet to be represented by the scale may be assumed at pleasure; in this case let it be 12·5 feet, and put x for the number of inches which will represent that distance; then—

$$\overset{\text{ft.}}{24} : \overset{\text{ft.}}{12{\cdot}5} :: 12 : x,$$

whence $x = \dfrac{12 \times 12{\cdot}5}{24} = 6{\cdot}25 =$ the number of inches required.

For by the question, 24 feet are represented by 1 foot, or 12 inches, and 24 feet evidently has the same ratio to 12·5 feet, that 1 foot (= 12 inches) has to the number of inches which will represent 12·5 feet.

Construction. (Pl. III. Fig. 7.) Draw, in pencil, three straight lines, rather more than $6\frac{1}{4}$ inches long, parallel to one another, and $\frac{4}{80}$ of an inch apart. On the lowest of these measure off a distance of 6·25 inches, and divide this distance into 5 equal parts, each of these will represent 2·5 feet, that is, 10 quarter-feet. Subdivide the left-hand primary division into 10 equal parts, to show single quarter-feet. Through each of the primary divisions draw perpendiculars from the lowest of the three lines to the uppermost. Through the secondary divisions draw perpendiculars to the middle line, and to half-way between the middle and upper lines alternately, as shown in the diagram. Ink the middle line of the three lightly, the bottom one rather heavily, the top one not at all; commencing from the left, number the secondary divisions 10, 8, 6, 4, 2, 0; the primary ones, 10, 20, 30, 40; opposite the last number print the words quarter-feet, and the scale is completed.

2. To construct a scale of 2 miles to an inch (Pl. III. Fig. 9).

Let the scale represent 11 miles: then—

$$\overset{\text{miles.}}{2} : \overset{\text{miles.}}{11} :: \overset{\text{in.}}{1} : \overset{\text{in.}}{5{\cdot}5} = \text{the length of the scale.}$$

Divide a line 5·5 inches long into 11 equal parts, each of these will represent a mile. Subdivide the first of these into 8 equal parts, to show furlongs. Complete the scale as shown in the figure.

$$\text{Representative fraction} = \frac{1}{2 \times 1760 \times 36} = \frac{1}{126720}.$$

3. **To construct a scale of 12 feet to ·875 inch (Pl. III. Fig. 11).**

Let the scale represent 60 feet: then—

$$\overset{\text{ft.}}{12} : \overset{\text{ft.}}{60} :: \overset{\text{in.}}{\cdot 875} : \overset{\text{in.}}{4 \cdot 375}$$

Divide a line 4·375 inches long into 6 equal parts, to show tens of feet, and the first of these into ten equal parts to show feet.

$$\text{Representative fraction} = \frac{\cdot 875}{144} = \frac{7}{1152}.$$

4. **The representative fractions of two plans of a Russian fort are $\frac{1}{800}$ and $\frac{1}{1260}$: construct a scale of French toises for the former and one of Russian archines for the latter.**

(1 toise=2·13142 English yards.)
(1 archine=·7777 English yards.)

(i.) **To make a scale of toises, $\frac{1}{800}$, 80 toises long.**

$$\overset{\text{toises.}}{800} : \overset{\text{toises.}}{80} :: \overset{\text{toise.}}{1} : \text{length of scale in toises.}$$

$$\therefore \text{length} = \frac{\overset{\text{English inches.}}{2 \cdot 13142 \times 36 \times 80}}{800} = \overset{\text{inches.}}{7 \cdot 673112}.$$

Divide a line 7·673 inches long into 8 equal parts, to show tens of toises; subdivide the first primary division into ten equal parts, to show toises.

(ii.) **To make a scale of archines, $\frac{1}{1260}$, 300 archines long.**

$$\overset{\text{ar.}}{1260} : \overset{\text{ar.}}{300} :: \overset{\text{ar.}}{1} : \text{length of scale in archines.}$$

$$\therefore \text{length} = \frac{\overset{\text{English inches.}}{\cdot 7777 \times 36 \times 300}}{1260} = \overset{\text{in.}}{6 \cdot 666} = \overset{\text{in.}}{6 \cdot 67} \text{ nearly.}$$

Divide a line 6·67 inches long into 15 equal parts, each of which will show 20 archines: subdivide the first of these into 4 equal parts, each of which will show 5 archines.

Obs. The length of these scales is optional. Every scale should, however, be sufficiently long to allow any lines in the drawing, except perhaps the very longest, to be measured at once.

COMPARATIVE SCALES.

When the scale of a drawing is adapted to one unit of length, it is sometimes necessary to construct another scale, in which the unit is different. These scales, called COMPARATIVE or CORRESPONDING SCALES, have the same representative fraction, and differ in graduation only.

If the unit of the proposed scale is a multiple, or a measure of the unit of the given one, the change is easily effected. When such is not the case, the length of the new scale may be determined as shown in the following examples.

Ex. 5. A scale of French leagues is attached to a map of France: the distance between two places, known to be 25 leagues apart, is represented on this map by 3·75 inches. Construct the corresponding scale of English miles.

(1 French league=4262·84 English yards).

Let the scale represent 100 miles, and put x for the number of inches in its length.

Then, 25 French leagues $= \frac{4262 \cdot 84}{1760} \times 25$ English miles.

$$\therefore \underset{\text{Eng. miles.}}{\frac{4262 \cdot 84 \times 25}{1760}} : \underset{\text{Eng. miles.}}{100} :: 3 \cdot 75 : x,$$

whence $x = \frac{3 \cdot 75 \times 100 \times 1760}{4262 \cdot 84 \times 25} = 6 \cdot 19$ nearly.

Divide a line 6·19 inches long into 10 equal parts, to show tens of miles; subdivide the first primary division into 10 equal parts to show miles.

Complete the scale as in the preceding examples (Pl. III. Fig. 12).

Ex. 6. To construct a scale of the Austrian fuss, comparative to *Ex.* 1. P. 41.

(1 fuss=·34568 English yards.)

Let the scale be 15 fuss long; then 15 fuss=·34568 × 15 × 3 English feet, and 24 feet are represented by 12 inches.

$$\therefore \overset{\text{ft.}}{24} : \overset{\text{ft.}}{\cdot 34568 \times 3 \times 15} :: \overset{\text{in.}}{12} : \text{length of scale in inches;}$$

whence the length $= \dfrac{\overset{\text{in.}}{12} \times \cdot 34568 \times 3 \times 15}{24} = \overset{\text{in.}}{7 \cdot 7778} = 7 \cdot 78$ inches nearly.

The scale is constructed as shown in Fig. 8, Pl. III.

$$\text{Representative fraction} = \frac{7 \cdot 7778}{\cdot 34568 \times 15 \times 3 \times 12} = \frac{1}{24}.$$

Ex. 7. A scale of Milanese miglios corresponding to *Ex.* 2. P. 61.

(1 miglio=1·0277 mile.)

The length of this scale will be determined in a manner differing somewhat from that adopted in the preceding examples, but based upon the same principles.

It is evident that 1 mile English has the same ratio to 1 miglio that a line which represents any number of English miles has to the line representing the same number of miglios. The proportion may therefore stand thus:—

$\overset{\text{Eng. mile.}}{1} : \overset{\text{Eng. mile.}}{1 \cdot 0277} :: \overset{\text{in.}}{5 \cdot 5}$: number of inches representing 11 miglios.

$$\therefore \text{length of scale} = \overset{\text{inches.}}{5 \cdot 5} \times 1 \cdot 0277 = \overset{\text{inches.}}{5 \cdot 65235}.$$

Had it been required to represent any other number of miglios, as, for example, 15, the proportion might have stood:

$$\overset{\text{miles.}}{11} : \overset{\text{miles.}}{1 \cdot 0277} \times 15 :: \overset{\text{in.}}{5 \cdot 5} : \overset{\text{in.}}{7 \cdot 70775} = \overset{\text{in.}}{7 \cdot 71} \text{ nearly.}$$

$$\text{or thus, } \overset{\text{miles.}}{2} : \overset{\text{miles.}}{1 \cdot 0277} \times 15 :: \overset{\text{in.}}{1} : \overset{\text{in.}}{7 \cdot 70775}.$$

The scale may be seen completed in Fig. 10, Pl. III.

Ex. 8. A scale of French kilomètres comparative to *Ex.* 2. P. 61.

(1 kilomètre=·62138 Eng. mile=1093·63 yards.)

miles. miles. in.
2 : ·62138 × 20 : : 1 : the number of inches representing 20 kilomètres=6·2138 inches.

DIAGONAL SCALES.

The scales, of which the construction and application have been explained, though very useful, are not well adapted for measuring minute distances. There is, however, another kind of scale, termed, from its construction, the DIAGONAL SCALE, by means of which this object can be accomplished with great accuracy.

For the principle upon which the construction of these scales depends see Prob. VI. Cor.; its application will be shown by an example.

Ex. To construct a diagonal scale to show hundredths of an inch full size.

Construction. Draw 11 equidistant parallel lines, $\frac{4}{60}$ of an inch apart (Pl. III. Fig. 13), and rather more than 7 inches long; call these lines horizontals, for convenience of reference. Draw a twelfth line at a little greater distance below the eleventh. Through eight points, in this last line, one inch apart, draw perpendiculars to meet the top line and cut all the others—call these verticals. Subdivide the left-hand primary division on the eleventh horizontal into ten equal parts. From the first point of subdivision on the left draw a straight line to the point in which the left-hand vertical cuts the top horizontal; through each of the other points of subdivision draw lines parallel to this line, meeting the first horizontal, and cutting all the others—call these lines diagonals. The scale being now constructed may be completed as shown in the figure.

Use.—Ex. 1. To take off 2·3 inches.

Placing the point of one leg of the dividers on the point in which the vertical 2 meets the uppermost horizontal, extend the

other leg to the point in which the diagonal 2 meets the same horizontal. The distance between the points of the dividers will be 2·3 inches, as required.

Ex. 2. To take off 3·47 inches.

Placing one leg of the dividers at the intersection of the vertical 3 with the horizontal 7, extend the other to the point in which the diagonal 4 meets the same horizontal.

This distance will be 3 in. + ·07 in. + ·4 in. = 3·47 in. For on the horizontal 7 the distance from the vertical 3 to the vertical marked 0 is 3 inches: from this vertical to the diagonal marked 0 is $\frac{7}{100}$ inch; from diagonal 0 to diagonal 4 is $\frac{4}{10}$ inch.

The same method may be applied to any other horizontal.

THE VERNIER.

Def. The vernier is a scale attached to the graduated limb of an astronomical or other instrument, for the purpose of measuring fractional parts of the divisions on the limb. It is connected with the limb in such a manner that, whilst remaining in contact with the graduated part, it can be moved smoothly along it. The description of this instrument does not fall within the scope of this work: since, however, vernier scales are sometimes used instead of diagonal scales, the principle of their construction will be explained.

Let L = the length of a subdivision on the scale;
V = the length of a subdivision on the vernier scale;
n = the denominator of the fractions required;
and assume, $nV = (n+1) L$.

Then $V - L = \frac{L}{n}$ that is, the difference between a subdivision on the vernier and one on the scale, is equal to $\frac{1}{n}$th of the latter.

Ex. Construct a scale of $\frac{1}{100}$ to show feet and tenths.

Let the scale be drawn in the ordinary way but subdivided throughout its entire length (Pl. III. Fig. 15); set off to the left, on the upper line, a distance equal to 11 subdivisions, com-

mencing from the zero: divide this into 10 equal parts as shown in the figure.

Let it be required to take off 26·7 feet.

Subtract 7 from 26, the remainder is 19: place one point of the compasses on the 19th subdivision on the upper line of the scale, and the other on the 7th division of the vernier.

The distance between the points will then be, 19 feet + 7 feet + the difference between 7 divisions on the vernier and 7 on the scale, which difference is from the construction evidently $\frac{7}{10}$ of a foot: consequently the required length 26·7 feet has been taken off.

A scale to measure feet and inches may be constructed in a similar manner.

EXAMPLES FOR PRACTICE.

1. Construct a scale of yards 5·5 in. to a mile, and give its representative fraction.
2. Construct a scale of paces for a plan drawn on a scale of 4 in. to a mile.
3. Construct a scale of 4 in. to a mile, and give its representative fraction.
4. Construct a comparative scale of paces. 1 pace = 30 in.
5. Construct a scale of feet to correspond with a French scale of 10 mètres to an inch. 1 mètre = 1·0936 yd.
6. Construct a scale of 8 in. to a mile, to measure yards, and give its representative fraction.
7. Construct a scale of paces 8 in. to a mile. 1 pace = 2·8 ft.
8. A map of a country has to be drawn to such a scale that 163 miles shall occupy, on paper, a distance of 13·2 in.; construct the scale so that single miles may be measured from it.
9. The distance between two prominent points of a fort is ascertained to be 1173·33 yds.; on a plan of the fort and the surrounding country, this distance is 4 in.; draw a scale of yards for the plan.
10. Construct a scale of yards having for its representative fraction $\frac{1}{5000}$.

11. Draw a scale of yards for a military sketch representing 25,000 ft. on the ground by 1 foot on the sketch.

12. Draw a scale to measure feet on a plan on which the distance between any two points is $\frac{1}{216}$ of the real distance.

13. Construct a scale of 125 ft. to an inch, and give its representative fraction.

14. Construct a plain scale of yards, $4\frac{3}{4}$ in. to a mile.

15. Draw a plain scale of yards: 5·5 in.=1700 yds. Give the representative fraction of the scale.

16. Construct a plain scale of 1700 yds. to 7 in., and print over it its representative fraction.

17. Draw a scale of $5\frac{1}{2}$ in. to a mile, to read to 10 yards.

18. Two plans of the same ground are drawn, one to a scale of $1\frac{3}{4}$ in. to a mile, and the other $1\frac{3}{4}$ mile to an inch. Show the length of the line which, for each plan, will represent $3\frac{1}{2}$ miles. Give also the representative fractions of the scales.

19. The French mètre is equal to 3·281 ft., but it may be considered, approximately for this question, equal to 39 in.; it is decimally divided into decimètres, centimètres, and millimètres. Construct a diagonal scale, (the proportional fraction being $\frac{2}{13}$) which shall represent mètres, decimètres, and centimètres.

20. A kilomètre is equal to 1000 mètres. On measuring a line on a French military map, I find that 2 in. in the map are equivalent to 5 kilomètres on the scale attached to it. Find the proportional fraction, and construct a comparative scale in miles and furlongs.

21. Construct a scale of $\frac{1}{1760}$.

22. A scale of 8·5 ft. to an inch, to measure single feet.

23. Draw a plain scale of $1\frac{2}{3}$ ft. to an inch, to show feet and inches. Give its representative fraction.

24. Construct a plain scale of yards 290 yds. to an inch, and also a corresponding scale of Russian sachines.

(1 sachine=2·332 yards.)

25. Construct a plain scale of 8 in. to a mile to measure yards; and a corresponding scale of chains.

1 chain=66 feet.

26. Draw a plain scale of Spanish yards to correspond with a scale of 6·5 in. to an English mile.

1 Spanish yard$=$·9277 English yard.

27. Draw a scale of inches 6 in. long, and divide the first inch division into ten minor divisions. Number its divisions and print over it 'Scale of Inches.'

28. A plain scale of yards is required for a military plan, on which a horizontal distance of 750 yds., on the ground, is represented by $3\frac{1}{2}$ in. Construct the scale.

Construct a comparative scale of Spanish yards.

29. Draw a scale of yards, $5\frac{1}{2}$ in. to a mile; and a comparative scale of paces. 25 paces$=$22 yards.

30. A scale of mètres $\frac{1}{2400}$. 1 mètre$=$1·0936 Eng. yd.

31. A scale of 22 yds. to an inch, 100 yds. long, on which single yards can be measured. Draw a corresponding scale of toises.

32. A scale of 5 miles to ·75 in., and a corresponding scale of Russian versts. 1 verst$=$1166·68 yds.

33. A scale of yards $\frac{1}{300}$, 40 yds. long, and a comparative scale of Russian archines. 1 archine$=$·7777 yd.

34. Construct scales of Bavarian rods and Bavarian ells corresponding to 11·5 ft. to an inch; the rod being equivalent to 3·1917 Eng. yds. and divided into 10 feet, and the ell being$=$·74845 Eng. yds.

35. An Englishman, wishing to examine a Spanish plan, finds only a scale of Spanish palms, 20 to an inch. Supply him with a corresponding scale of English feet, taking the palm as ·684 English ft. Show 50 feet (Pl. III. Fig. 14).

36. The distance between two towns on a Swedish map is 9·125 in. English, they are 120 Swedish miles apart. Construct for this map a scale of English miles.

1 mile Swedish$=$6·6412 English miles.

37. Finding that the distance between two points on a Swedish map is 7 in., and the real distance on the ground 5000 alners, construct for it a scale of feet, the alner being ·6493 of an English yard.

38. Draw a diagonal scale to show 1000ths of a foot full size.

39. A Prussian fathom contains 6 Rhenish feet, each 1·0297 English feet. Construct a scale of fathoms $\frac{3}{700}$ showing feet diagonally.

40. Make a scale of 10 ft. to 1·5 in. showing inches diagonally, and explain the principle of construction.

MISCELLANEOUS EXERCISES.

1. Draw two circles having radii of 1 and 1·2 in. respectively, touching each other (*Prob.* VII.).

2. Construct a scale of miles and furlongs having for its representative fraction $\frac{1}{63360}$.

3. Draw a straight line, 3·25 in. long, and from its extremity draw a perpendicular by construction, without adding to the length of the line (*Prob.* V. 4).

4. Describe a circle, of ·75 in. radius, and describe about it a triangle, two of whose angles are 53° and 38° respectively.

5. Draw three parallel lines at $\frac{1}{8}$ in. apart, and 2 in. long. The first line to be a dark line, the second a line of medium thickness, and the third a fine line.

6. Construct a scale of yards 5 in. to the mile, and give its representative fraction.

7. Construct a triangle having its sides A B, B C and C A =2·5, 3·5 and 4 in. respectively; and make an angle equal to the angle A B C (*Prob.* II.).

8. Draw two straight lines each about 2 in. long, and inclined to each other at an angle of 100°. Draw a circle of 1 in. radius touching both of them (*Prob.* XIII.).

9. Draw, on a scale of 60 yds. to an inch, a triangle having two sides equal to 180 yds. and 240 yds. respectively, and the angle included between them equal to 75°. Ascertain accurately and figure the length of the third side and the values of the remaining angles. The triangle to be penned in with a very thin line.

10. Draw two parallel lines with a straight edge and compasses. Let the lines be about 3 in. long, and $\frac{3}{4}$ in. apart.

11. Construct, on a scale of 45 ft. to an inch, a figure A B C D E F, according to the following instructions:—

A B=135 ft.; B C=117 ft.; the angle A B C=135°; C D=88 ft.; the distance A D=202 ft.; D E=141 ft.; distance A E=148 ft.; E F=84 ft.; F A=87 ft.

Ink in the figure A B C D E F in a dark line: the lines A D, A E in chain dots.

12. Draw five figures similar to A B C D E F, and inside it, at distances from it of 2, 4, 6, 8, 10 ft. respectively. Ink in these five figures in fine and dotted lines alternately, commencing with a fine line for the innermost figure. Print the letters distinguishing the lines of the external figure, and figure the plan according to the dimensions given in *Ex.* 11.

13. Construct a scale for the figures drawn in *Ex.* 11 and 12, and print neatly above it 'Scale of 45 ft. to an inch.'

14. Assume three points A, B, C, such that A is $2\frac{1}{2}$ in. from C, and $1\frac{1}{2}$ in. from B, B being $1\frac{3}{4}$ in. from C. Describe a circle passing through the points (*Prob.* X.).

15. Draw a straight line; from a point very close to, but within, the right-hand end of the line, raise a perpendicular (*Prob.* V. 4).

16. From any point of the above perpendicular draw a line towards the original line, and at an angle of 30° with the perpendicular; and state at what angle it will meet the original, or base line.

17. The scale of the ordinary English barometer is extended up to 31 in. Construct a vernier scale to show the reading 30·65 in.

18. On a Chinese map, a distance called 17 li, measures 6·3886 in. Give the representative faction, and construct corresponding scales of English miles and French kilomètres.

1 Chinese li=·355871 English mile.
1 kilomètre =·62138 „

19. Construct a plain scale of 19 English ft. to an inch, and make a corresponding scale of Chinese Tehé.

1 Tehé=1·05 English ft.

20. Draw the scale which you would adopt in order that the

plan of a district 17 miles long and 13 miles wide might be made on a sheet of paper 10 in. long and $7\frac{1}{2}$ in. wide. Divide the scale so that you would be able to measure miles, furlongs, and chains.

21. Inscribe an octagon in a square, the side of the square being 2·43 inches.

22. One side of a triangle is 2·7 in. long, the opposite angle B is 65°, $\frac{b}{\sec A}=1{\cdot}6$ in. Construct the triangle.

23. On a base of 3 in. describe an irregular figure of 7 sides; reduce it to a triangle of equivalent area, and calculate that area.

24. Draw arcs of radii 2 and 3 in. respectively, with their centres 6 in. apart, and a third arc of 2·5 in. radius to touch the others externally.

25. Draw a line 2 in. long perpendicular to a given line 3 in. long, from a point $\frac{3}{4}$ in. from one end, by geometrical construction (*Prob.* V. 2).

26. Construct an isosceles triangle equal in area to the sum of 4 squares, of which the sides are respectively $\frac{1}{2}$, $\frac{5}{8}$, $\frac{7}{8}$, $1\frac{1}{4}$ inch.

27. Draw six parallel dotted lines of equal thickness, each 3·14 in. long, and at equal distances of $\frac{5}{16}$ in.

28. A right-angled triangle has a base of 2 in. and an area of 3 square in.: construct it, and also one similar to it of half its area (*Prob.* XXIII. *Cor.*).

29. The distance between London and Chatham is 30 miles, and measures on the map 18·3 in.; draw the scale of the map divided into miles and furlongs, and mark the representative fraction.

30. A map is 36 in. long and 30 in. broad, it contains an area of 25 acres; draw a scale for the map to show poles, yards, and (diagonally) feet.

31. Divide a straight line 5·3 in. long into 8 equal parts, and through the points of division draw parallel straight lines, $\frac{1}{2}$ in. apart, making them alternately dotted and continuous (*Prob.* XII.).

32. Construct 3 squares the areas of which are ·81, 1·96 and 2·56 square in.; and construct, geometrically, a fourth square, the area of which shall be equal to the sum of the areas of the other three (*Prob.* XXX.).

33. Construct a triangle having its sides equal to 2, $2\frac{1}{2}$, and 3 in. respectively, and find a point equidistant from its sides. Circumscribe a circle about the triangle (*Prob.* X.).

34. Draw a scale of Milan miles $\frac{1}{20000}$.

The Milan mile = 1·0277 English mile.

35. The distance between two points 1 Austrian mile apart, is represented on a map by 2·66 inches.

Construct a diagonal scale of English miles for the map, to measure hundredths of a mile. The Austrian mile = 3·3312 English miles.

36. Describe an arc passing through the three points A, B, and C, which are so situated that the distance from A to B is 1·75 in., from B to C 1·34 in., the lines joining A B and B C making with each other at B an angle of 133° (*Prob.* X.).

37. The distance which A B in *Ex.* 36 represents is 365 yards. Construct a scale of yards for the diagram.

38. On an Italian military plan, I find that the distance between two points measures 3 English inches, and on a scale attached to the plan 220 canne. Draw a comparative scale of English yards. 1 canna = 2·3008 English yards.

39. On a base, 3 in. long, construct an equilateral triangle, and in it inscribe a circle. (*Bisect two of the angles.*)

40. Draw an irregular hexagon of which the longest side shall measure 3 in. and the shortest side 1 in.; and construct a similar figure the longest side of which shall measure 2 in. (*Prob.* XX.).

41. Construct a scale to measure feet and inches on a drawing in which 1 in. represents 7 ft.

42. From a point A, the angles between the points B and C, and C and D, were observed to be 40° and 50°, the lines joining B C and C D being 1200 and 1500 yards long respectively, and forming at C, on the side nearest A, an angle of 155°. Find the point A. Scale 600 yds. to 1 in.

43. Construct a vernier scale, 8 ft. to an inch, to measure inches by means of the vernier.

44. Draw a straight line $4\frac{1}{2}$ in. long and divide it, by construction, into 5 equal parts (*Prob.* VI.).

45. Construct a triangle having two of its sides equal to 3 in. and $3\frac{1}{4}$ in. respectively, and the angle subtended by the longer side equal to 50°.

46. Draw a straight line 2 in. long, and on it describe the segment of a circle which shall contain an angle of 55° (*Prob.* IX.).

47. Draw two lines $2\frac{1}{2}$ in. apart and unite them by two curves of contrary flexure, having radii of $\frac{3}{4}$ and $1\frac{1}{4}$ in. respectively, in such a manner that the curves shall be tangential to each other, and each one of them tangential to one of the two straight lines (*Prob.* VIII.).

48. Draw a straight line about 2 in. long, and describe a quadrant of a circle with a radius of 1 in., touching it at one extremity (*Prob.* VIII.).

49. Construct a triangle of which the sides are as 4, 5, and 6, and about it describe a circle (*Prob.* X.).

50. Draw two parallel lines 5 in. long and $2\frac{1}{2}$ in. apart, and describe two circles between them touching the lines and each other (*Prob.* VIII.).

51. Divide a line, 6 in. long, in the proportion of the numbers 2, 3, 4, 5, 1 and 2, the divisions running in accord with the numbers as regards sequence. Ascertain and figure the length of each division in inches and decimals of an inch.

52. From one extremity of a line 3 in. long draw a perpendicular 2 in. long, without producing the line. Base the construction on geometrical principles (*Prob.* V. 4).

53. Find, by construction, a mean proportional between two lines 2·4 and 3·8 in. long, respectively (*Prob.* XI.).

54. A line 5 in. long is divided into 6 equal parts; draw parallel lines ·5 in. apart through the divisions of the given line.

55. Construct a square of 5·36 in. area, without extracting the square root of 5·36 (*Prob.* XI. *Cor.*).

56. Construct a square of which the area shall be equal to the sum of 4 squares having their sides ·5, 7·5, ·875, and 1·125 in. respectively (*Prob.* XXX.).

57. Divide a line 3 in. long into 7 equal parts (*Prob.* VI.).

58. Describe a square equal to the difference of two squares whose sides are 3·25 and 1·94 inches (*Prob.* XXIV.).

59. Make a triangle of which the sides are 3·5, 1·75, and 2 in. respectively. Describe a rectangle equal to the triangle (*Prob.* XVI.)

60. Describe a rectangle of which the sides are 3·45 and 2·65 in.; find its area.

61. Find a line which shall have the same ratio to a line 1·5 in. long, that 3 in. has to 1·75 inches (*Prob.* XXIX.).

62. Trisect a right angle.

63. Describe upon a line, 2 in. long, as a base, an isosceles triangle having its vertical angle equal to $\frac{1}{3}$ of a right angle (*Prob.* XXII.).

64. Draw parallel lines one inch apart through points in a straight line at distances of 2 inches (*Prob.* XII.).

65. Draw a circle circumscribing a triangle, of which the sides are respectively 4, 5, and 6 inches (*Prob.* X.).

66. Construct a square equal to a triangle, of which the sides are respectively 1·5, 2, and 2·25 inches (*Prob.* XVI. and XI.).

67. A B is a straight line 2 in. long; find, with the compasses only, a point P, in the continuation of A B, on the side of B, and 2 in. from it (*Euc.* IV. 15).

68. Reduce an irregular figure of five sides to an equivalent triangle, and calculate the area (*Prob.* XIX.).

69. In the triangle A B C, A B=150 yds., B C=180 yds., A C=250 yds. Find by construction a point P when the angles A P B and C P B are respectively 38° and 52° (*Prob.* XXV.).

70. Construct an equilateral triangle $2\frac{1}{2}$ in. high.

71. Describe an arc of 45° with a radius of 2 in., and bisect it.

72. Describe a circle of 1·85 in. diameter; assume a point 1·5 in. without it, and from this point draw a tangent to it (*Prob.* VIII.).

73. Draw parallel lines $4\frac{1}{2}$ in. long at $\frac{1}{8}$ in. apart. The first line to be a dark line; the second a fine line; the third a simple dotted line; and the fourth a chain dotted line.

74. Construct a square of 1·5 in. side, and describe about it a circle in dots: describe a second circle in a dark line concentric with the first, and $\frac{1}{10}$ in. outside of it (*Prob.* XXIII.).

75. Draw 4 concentric circles with radii=1 in., $1\frac{1}{10}$ in., $1\frac{1}{8}$

in., $1\frac{1}{4}$ in. respectively. Ink them in, commencing at the inner circle, with a dark line; as fine a line as you can draw; a moderately thin line, such as is suitable for the thin lines of an ordinary working drawing; and a dotted line.

76. Draw 3 circles each with a diameter of 2 in., so that each circle may touch the other two (*Prob.* VII.).

EXAMINATION PAPERS.

College of Preceptors.

Midsummer 1871.

1. Draw a line perpendicular to a given line at one extremity.

2. Draw a line about 2 in. long, divide it into any number of unequal parts; draw another line two-thirds the length of first, and divide it proportionally to it.

3. Construct a hexagon upon a given base line about half an inch long.

4. Construct a nonagon upon the same or a similar base line.

5. Describe an ellipse, the long diameter of which is half as long again as the short one, and draw a short line perpendicular to the curve of the ellipse at any point upon it.

Christmas 1871.

1. Construct an equilateral triangle, the altitude of which is 2 in.

2. Draw any triangle, and construct another precisely like it, by measuring the angles.

3. Give a method by which any regular polygon may be constructed, one side being given.

4. Construct a triangle, the sides of which are respectively 2 in., $1\frac{1}{2}$ in. and 3 in.; inscribe a circle within the triangle.

5. About the triangle, given in the last question, describe a circle.

Midsummer 1872.

1. Take two points, A and B, $1\frac{1}{2}$ in. apart and construct a circle of 1 in. radius passing through them.

2. On a base line, A B, 3 in. long, construct a triangle having the angle at $A=45°$, and at $B=60°$.

3. On a line 2 in. long construct a square, and find the size of another square half the area of the first.

4. Within a circle of $1\frac{1}{2}$ in. radius show a method by which any regular polygon may be constructed.

Christmas 1872.

1. Construct a rhomboid, one diagonal being 2 in., and adjacent sides $1\frac{1}{2}$ in. and 1 in. respectively

2. Draw a regular heptagon of 1 in. side, by a method applicable to any polygon.

3. Draw an ellipse, the long diameter of which is 2 in. and the short diameter $\frac{1}{2}$ in. Find the centre and axis or transverse and conjugate diameters of a given ellipse, the circumference only being given.

4. Inscribe three equal circles in an equilateral triangle, each circle touching two sides of triangle.

5. Draw a circle 3 in. diameter, and inscribe within it three equal circles.

Cambridge Local Examinations.

A.

1. Draw an ellipse, of which the conjugate diameter is 2 in., and the transverse 3 in. in length.

2. On a line 1 in. long construct a regular pentagon.

3. About a circle of 1 inch radius construct a regular hexagon.

4. Within a circle, 3 in. in diameter, place 5 equal circles touching one another, and the circumference of the given circle.

5. Construct a rectangle 2 in. by 1 in. and on one of the longer sides an isosceles triangle of equal area.

6. Construct an equilateral triangle of 2 in. sides, and within it inscribe a square.

B.

7. Construct an isosceles triangle, of which the base shall be 1 inch long and the opposite angle 30°.

8. Within a circle 3 in. in diameter place a cinqfoil of tangential circular arcs.

9. Find a mean proportional between two lines, $1\frac{1}{4}$ and $2\frac{1}{4}$ in. long.

10. Draw three circles of $1\frac{1}{2}$ in. diameter, each one touching the other two.

11. Within a circle $2\frac{1}{2}$ in. in diameter inscribe a rectangle, of which the longer sides are 2 in.

12. Construct an equilateral triangle of $1\frac{1}{2}$ in. sides and a rectangle of equal height and area.

Cambridge Local Examinations.

A.

1. Construct a regular pentagon, of which the sides are $1\frac{1}{2}$ in. long.

2. Draw the curve of an ellipse of which the longer diameter is $3\frac{1}{2}$ in. and the shorter $2\frac{1}{2}$ in. long.

3. Construct a rhombus having its sides 2 in. long and one angle 60°, within it inscribe a circle.

4. Within a circle, 3 in. diameter, inscribe three circles equal to and touching one another, and also touching the circumference of the given circle.

5. At one end of a line 2 in. long erect a perpendicular to it, and trisect the right angle.

6. Construct a square equal in area to an equilateral triangle of 1 in. side.

B.

1. Construct a regular decagon contained within a circle of 1 in. radius.

2. Within a rectangle 3 in. by $2\frac{1}{2}$ in. place a similar figure of which the longer side is $2\frac{1}{2}$ in. The smaller rectangle to have the same centre as, and its sides parallel to those of, the larger one.

3. About an equilateral triangle of 1 in. side describe a circle.

4. Find a mean proportional between two lines which are $1\frac{1}{2}$ and 2 in. long.

5. Draw 3 circles 1 in. in diameter each, touching the other two.

6. Construct an isosceles triangle having an angle of 30° opposite the base.

A.

1. Upon a line 1 in. long describe a regular hexagon.

2. Within a rectangle 3 in. by $2\frac{1}{2}$ in., inscribe an ellipse.

3. Draw a cinqfoil of adjacent semicircles of $\frac{1}{2}$ in. radius.

4. Within a triangle having sides 2, $1\frac{1}{2}$, and $1\frac{3}{4}$ in., inscribe a square.

5. Construct a square having an area equal to the rectangle in *Ex.* 2.

6. Draw an undulating continuous curve of 6 arcs of circles, each containing 90° and described with a radius of 1 in.

B.

1. Within a circle of 1 in. radius inscribe a regular octagon.

2. A rectangle has sides of $2\frac{1}{4}$ and $1\frac{1}{2}$ in.; give, by construction, a similar rectangle, of which the longer side is $1\frac{3}{8}$ in.

3. Draw a quatrefoil of tangential arcs of circles of $\frac{1}{2}$ in. radius.

4. Within a rhombus, of which the diagonals are 3 in. and $2\frac{1}{2}$ in., inscribe a circle.

5. Construct a regular pentagon of 1 in. side, and about it a second pentagon of $1\frac{1}{2}$ in. side, having the same centre and its sides parallel to those of the first figure.

6. Within a circle of $2\frac{1}{2}$ in. diameter place 3 equal circles, touching one another and the circumference of the given circle.

City of London College.

1. Describe an equilateral triangle of $3\frac{1}{4}$ in. altitude.

2. Divide a straight line $5\frac{3}{7}$ in. long into parts as the numbers 3, 7, 9, 10.

3. Describe an equilateral and equiangular pentagon on a line $1\frac{3}{4}$ in. long.

4. Inscribe a regular undecagon in a circle of $1\frac{4}{5}$ in. radius.

5. Draw an ellipse, of which the axes are 3 and $2\frac{1}{2}$ in. respectively.

6. Divide a pentagon into 4 parts, as 3, 5, 7, 9, by straight lines drawn from one of its angles.

7. Divide a triangle of which the sides are $2\frac{1}{2}$, 3, and 4 in. respectively, into 3 equal parts by lines drawn through the middle point of the longest side.

8. On a line 3 in. long describe a Gothic arch on an equilateral triangle.

Royal Military Academy, Woolwich.

1. In the triangle A B C, A B=2 in., B C=2·4 in., the angle A B C=52°. Bisect A B C by B D (D being a point in A C), and divide B D into 5 equal parts.

2. Draw the regular nonagon A B C D E F G H K with a side of 1·7 in. Reduce it to a triangle of equal area having E for its vertex and A B produced for its base.

3. Draw a semi-ellipse, of which the major axis is 4 in. and the minor axis 2 in.

4. Find a mean proportional and a third proportional to two lines 1 in. and 1·69 in. long.

5. Draw circles with radii of 1 in., $1\frac{1}{2}$ and 2 in. to touch each other externally: join the centres of the circles; and to the resulting triangle apply the inscribed and circumscribed circles.

6. Divide a line of 3·75 in. into two segments, such that the rectangle contained by them may be 2 in. in area.

7. Construct a regular pentagon of 7 in. area.

8. Construct two squares, one double the area of the other, the area of the greater being 5 inches.

9. Draw four lines meeting in a point, so that a line drawn parallel to any one of them may be divided into two equal segments by the other three.

1. Make a regular pentagon, side 2·5 in., and draw any diagonal; construct an equilateral triangle equal to the larger portion of the polygon so divided.

2. Through one angle of a regular octagon of 1·8 in. side draw a line to cut off $\frac{2}{7}$ths of the figure.

3. Draw a circle of 2 in. radius, and through a point 4·5 inches from the centre, draw a line cutting off from the circle a segment containing an angle of 30°.

Construct a scale of furlongs $\frac{1}{63360}$, and divide it diagonally to show chains.

4. Two buoys in a harbour are distant from each other 15 kilometres; on a drawing of the harbour their plans are 8·5 inches apart; required a scale of English miles for the plan.

1 kilometre = 1093·33 yards.

5. A line on a plan, 10·85 inches long, represents a length of 83·5 feet; a second plan of the same work is to be drawn, so that the area of the second shall be to that of the first as 3 : 2; draw the scale for this second plan to show feet and inches.

6. Divide a line 1·2 in. long into 77 equal parts by diagonal division.

7. Circumscribe a triangle having its sides as 2 : 3 : 3·5 about a circle of 2 in. diameter.

8. Draw a regular pentagon of 5 inches area, and construct a regular hexagon of an equal area.

9. Inscribe 3 equal circles in one of 1·5 in. radius, each touching the other two.

10. A point H is 2 in. from a line A B; draw the curve the distance of every point of which from H and A B is in the constant ratio of 3 : 4.

1. Through two points 4 in. apart, draw lines converging to a point without the paper, and through any assumed point (P) another line to pass through the intersection of the first pair.

2. Draw a circle through point (P) in question (1) touching the two lines first drawn.

3. Construct a regular nonagon of 4·5 inches diameter, and an equilateral triangle equal to $\frac{4}{5}$ths of the area of the polygon.

4. The semi-conjugate diameters of an ellipse are 2·5 and 1·3 in., respectively, and the included angle is 63°; draw the curve, and determine its axis.

5. Given, point T, in a parabola, and its distance F T from the focus F 1·6 inehes, the line F T making an angle of 31°, with a tangent to the curve at the point T, to determine the axis, directrix, and curve.

6. Make a scale of feet $\frac{1}{84}$ to show inches diagonally.

a. The mouth of a harbour is 1250 yards wide, and this width is expressed upon a plan by 15·5 inches; required a scale of French mètres for the drawing.

7. The area of a rectangular piece of ground is 92,160 square links. In a drawing it occupies an area of 19·6 square inches. The sides of the rectangle are in the ratio of 5 to 8. Make a

scale of chains showing 4 chains, and divide diagonally to show links.

1 chain = 100 links.

8. A regular octagon has a diameter of 4·5″; through a point in its perimeter $\frac{1}{8}$th of an inch from an angle, draw a straight line cutting off $\frac{4}{7}$ths of the figure.

9. Construct an equilateral triangle with an area of 6 inches.

10. A trapezoid has its parallel sides 2″ and 5″, these latter being 3″ apart; construct an isosceles triangle equal to the figure and with same altitude: or an equal triangle having its sides as the number 4, 5, 7.

11. Four sectors of circles have angles of 60° and radii 2″, 2·5″, 3″, and 4″; construct a 'similar sector' equal to the four.

12. Construct the quantity $\sqrt{a^2+5\ b^2}$ when $a = 2''$, $b=1{\cdot}5''$.

13. Points D and E are 2″ apart; the straight line D E is produced 2″ to F to make an angle D F G=35° with F G: through D and E draw a circle to touch the straight line F G.

14. Make a diagonal scale of French mètres $\frac{1}{360}$ to show decimètres.

1. Divide a line, 4″ long, into two segments such that the rectangle contained by them may be 3 square inches in area.

2. Construct a square of 5 in. area, and an equilateral triangle equal to it.

3. Inscribe a square in an isosceles triangle of 2·5″ base and 3″ side.

4. Construct a triangle similar to the last, but of double its area.

5. Bisect this last triangle by a line parallel to its shortest side.

6. Draw a scale of $\frac{1}{100}$ to show chains and links. Chain=22 yards.

7. Divide a square of 2·5″ side into three equal areas by lines drawn through one corner.

8. The radii of two circles are 1″, 1·75″; cut off from the circumference of the greater an arc equal to half that of the lesser. Calculation must be used here.

9. Inscribe a rectangle of 5 in. area in a circle of 2″ radius.

10. Construct a rectangle of 6 in. area, its sides being as 3 : 2.

11. Construct the following quantities $\sqrt{8}$, $\sqrt{\frac{2}{3}}$, $\frac{\sqrt{5}-1}{2}$, $\sqrt{7+3}$.

12. Construct a scale of paces of 32″ of $\frac{1}{90}$.

13. Two sides, A B and B C, of a triangle are 3″ and 1·5″, and they include an angle of 70°: through point B draw a straight line to divide the third side A C:—

(1) In the ratio of A B to B C.
(2) „ of A B+B C to B C.

14. If A and C, in preceding question, are the foci of an ellipse, and B a point in the curve, draw the ellipse: and a tangent to it at the point B.

(Only half the curve need be shown, correctly drawn.)

15. An isosceles right-angled triangle has its equal sides 3″ each: required an equilateral triangle equal to it.

16. Trisect a square, side 2·5″, by straight lines drawn through one angle.

17. Give geometrical constructions of the following identities: When $a = 2''$ $b = \cdot 7''$.

$$a^2+b^2 = 2\left\{\left(\frac{a+b}{2}\right)^2+\left(\frac{a-b}{2}\right)^2\right\}$$

$$ab+\left(\frac{a-b}{2}\right)^2=\left(\frac{a+b}{2}\right)^2$$

18. Make a diagonal scale of yards comparative to one of Russian versts of $\frac{1}{1760}$.

19. The sides of a rectangle are 6″ and 2″: determine by a single construction the sides of 4 equal rectangles having their containing lines in different ratios.

R.M.C. Sandhurst Papers.

I.

1. Draw 9 circles of $\frac{3}{4}$ in. radius in three rows, their centres to be in lines parallel to each other and $1\frac{1}{2}$ in. apart.

2. Draw a circle with a radius of $\frac{3}{4}$ in. Inscribe and circumscribe this circle with two equilateral triangles having their sides parallel.

3. Find a mean proportional between two lines $2\frac{1}{4}$ in. and $\frac{1}{4}$ in. long, and figure its length on the diagram.

4. Draw a regular hexagon, of $1\frac{1}{2}$ inch side, and within it

six equal circles, each touching two other circles and also two sides of the polygon.

5. Draw a circle with a radius of 1 in., and place in it a chord such that the angle standing on it and having the point in the circumference may be 47°.

II.

1. Draw a scale of a foot to an inch, to show inches and feet.

2. It is desired to increase the size of part of a plan, which is drawn to a scale of $\frac{1}{25440}$, to one which is to be on a scale of 8 feet to an eighth of an inch. Draw the scales.

3. Construct a triangle, two of whose sides shall be 2·4 and 3·15 inches respectively, and the included angle 136°.

4. Draw a circle, in whose circumference the three angles of a triangle shall lie, whose sides shall be in the proportion of 6, 19, and 21. Make the diagram on a suitable scale.

5. Draw a regular pentagon of $2\frac{1}{2}$ inches side, and convert it into a regular hexagon of equal area.

6. Determine, by construction, a third proportional to two lines of $1\frac{1}{2}$ and 3·625 inches in length, greater than either of them.

7. In a circle of 2·8 inches radius inscribe 5 equal circles.

8. Draw the straight line A B, 6 inches long, and take a point C $\frac{1}{2}$ an inch above A B and 2 inches from A; through A C B draw the arc of a circle (the centre being inaccessible), determining at least 4 other points in the arc.

III.

1. Trisect the line A B which is 5·3 inches long in the points C and D; at A and B erect perpendiculars by different methods without in either case producing A B; at C draw a line C E, making the angle A C E = 105°; and at D draw a line D F making the angle B D F = 70°; constructing the angles in each case (i.e. not measuring them by means of the protractor or sector).

2. Draw a circle with a radius of $2\frac{3}{4}$ inches. In it inscribe a triangle, having its sides in the proportion of 3, 4, and 5. Write down the magnitudes of the sides and angles of the triangle.

3. About a circle of 1·7 inch radius, describe a regular hexagon.

4. Draw a scale to measure distances between 700 yards and 10 yards, in which 250 yards are represented by 2·25 inches.

5. From the above scale construct an isosceles triangle, of which the altitude is 380 yards, and each of the equal angles 35°. Write down the lengths (in yards) of the sides and base of the triangle.

6. Construct a mean proportional between two lines $1\frac{1}{4}$ and $1\frac{4}{5}$ inches long; and a fourth proportional to the same two lines and the mean proportional, taken in the order in which they are written.

IV.

1. Draw the line A B $3\frac{1}{4}$ inches long; from A and B draw (on the same side of A B) lines meeting in C, making the angles C A B, C B A, 110° and 40° respectively (these angles are to be *constructed*, *i.e.* not measured by either the protractor or the sector). From C draw a perpendicular to A B produced, meeting it in D. Write down the lengths of A C, B C, and C D.

2. In an equilateral triangle of 5·2 inches side, inscribe 3 equal circles. Each circle is to touch each of the other circles and *two* sides of the triangle.

3. Construct a regular hexagon with a side of $2\frac{3}{4}$ inches, and in it inscribe a circle.

4. Construct a parallelogram with sides of 3·9 and 2·7 inches, one of the angles measuring 65°. Divide the parallelogram into 6 equal parallelograms by means of lines parallel to the shorter sides.

5. In the straight line A B 350 feet long, take points C and D, making A C and A D 95 feet and 230 feet long respectively. At C, D, erect (by construction, and on opposite sides of A B) perpendiculars C E, D F, 135 feet and 110 feet long respectively. Join A E, E F, F B.

Measure and write down the lengths (in feet) of these lines and the size of the angles A E F, E F B.

Scale (which need not be drawn) 60 feet = 1 inch.

6. Draw a scale of feet long enough to measure 500 feet, the least dimension being 10 feet; 60 feet being represented by ·75 inches; give the calculation.

7. Draw a diagonal scale, of $\frac{3}{4}$ of a mile to the inch, to measure miles, furlongs, and chains, 4 miles to be the greatest length shown. Give the calculation, and mark the representative fraction. From the scale draw a line 2 miles, 2 furlongs, 2 chains long.

N.B.—1 mile = 8 furlongs, 1 furlong = 10 chains.

V.

1. Draw a square whose side is 3·25 inches.

2. Draw a circle 3·4 inches in diameter. Find its centre by means only of a ruler whose edges are parallel.

3. Draw two circles of radii 8 inches and 1·2 inches with centres 2·5 inches apart. Draw an arc of a circle 5 inches radius touching internally these circles.

4. Draw an equilateral hexagon 1 2 inches side. Reduce the figure to a triangle and ascertain its area.

5. Find a mean proportional to the lines whose lengths are 2·5 inches and 3·6 inches, and give its length.

6. Divide a line A B, 2·7 inches long in the point C, so that the rectangle under A B and B C shall equal the square on A C.

7. Construct a scale of chains. Representative fraction $\frac{1}{10560}$. Smallest unit 1 chain. Show 1 mile.

N.B.—One chain = 66 feet.

VI.

1. Draw a parallelogram of which the adjacent sides measure $2\frac{7}{8}$ and $3\frac{3}{4}$ inches respectively and two of the angles 57° each. Divide the parallelogram into 16 equal parallelograms similar to the original figure.

2. Determine by construction a mean proportional to two straight lines 1·6 and 2·5 inches long, and a 4th proportional to these two lines and the mean proportional. Write down the lengths of these proportionals.

3. Upon a straight line $3\frac{3}{8}$ inches long, construct a square; bisect the line, and upon one of the halves construct a second square inside the first one. Reduce the space by which the first square exceeds the second to a triangle of equal area.

4. From a point B in the line A B, 52 feet long, draw B C 39 feet long, making the angle A B C 125°, from C draw C D (by

construction, and without producing B C), 35 feet long perpendicular to B C; from D draw D E, 43 feet long, making the angle C D E, 75°. Join A E. Write down its length in feet and the size of the angles A E D, B A E. A B C D E forms an irregular five-sided figure. Scale (which need not be drawn) 15 feet = 1 inch.

5. Draw two circles with radii of $1\frac{1}{4}$ and $1\frac{3}{4}$ inch respectively to touch each other externally, and an arc with a radius of $2\frac{1}{2}$ inches, touching externally the first two circles.

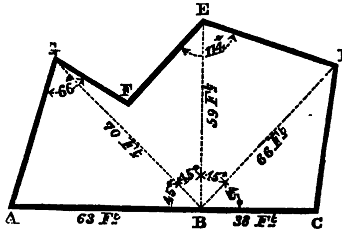
6. Draw a plain scale of miles and furlongs in which $\frac{3}{4}$ of a furlong is represented by $\frac{1}{16}$ of an inch. The scale should be long enough to measure 10 miles. Show the calculation, and mark the representative fraction.

7. Draw a diagonal scale, 850 paces to the inch, to measure all distances between 10 and 6,000 paces. From the scale draw a line 3,560 paces long. Show the calculation, and mark the representative fraction, assuming the pace to measure 32 inches.

VII.

1. In a circle having a radius of 2·3 inches place a triangle having its sides in the proportion of 4, 5, and 6. In the triangle inscribe a circle.

FIG. 27.



2. Divide by construction, AB $3\frac{5}{8}$ inches long, into three equal parts in C and D. At A construct a perpendicular to A B $2\frac{1}{3}$ inches long. At C and B draw C E and B F $3\frac{1}{4}$ and $2\frac{7}{8}$ inches long respectively, making the angles E C B and F B A (by construction) 60° and 135° respectively.

3. On a straight line 1·9 inches long construct a regular heptagon. Reduce the figure to a triangle of equal area.

4. The accompanying diagram (fig. 27) gives the dimensions, &c. of the irregular figure A C D E F G. Draw the figure to the given scale. Measure and write down the length (in feet) of E F and the magnitude of the angles G A C and C D E.

Scale (which need not be drawn) 20 feet=1 inch.

5. Construct the segment and sector of a circle having each the same chord, 3·4 inches, and the same radius, 2·3 inches. Measure and write down the magnitude of the angle in the segment.

6. Draw a plain scale of yards and eighths of yards long enough to measure 7 yards, $\frac{5}{8}$ of a yard being represented by ·45 inches. Give the calculation and mark the representative fraction.

7. Draw a diagonal scale of furlongs and chains, long enough to measure 50 furlongs, 23 furlongs being represented by 2·7 inches. From the scale draw a line 32 furlongs 7 chains long. Give the representative fraction and the calculation.

R.M.A. Woolwich Entrance Papers.

I.

1. Draw a straight line A B 3·2 in. long; bisect it in C; take points D and E in A B, one on each side of C, 1·4 in. and ·8 in. from it; on D E describe a square.

2. Construct a triangle A B C, having A B 3·6 in., B C $2\frac{1}{2}$ in., and the angle A B C 115°. Divide A B into 5 equal parts, and through each point of division draw lines, alternately, dotted and chain dotted, parallel to A C, terminating in B C. Measure and write down the size of the angles B A C and A C B, and the length of each of the parallels.

3. Describe a circle with a radius of $1\frac{1}{2}$ in., and two other circles with radii of 1 in. and $\frac{7}{8}$ in., touching the first circle respectively, externally and internally, in the same point. Decribe a fourth circle with a radius of $1\frac{1}{4}$ in., touching the two smaller ones.

4. Divide a straight line 3·3 in. long into two parts, so that

the rectangle contained by the whole and one of the parts shall be equal to the square of the other part.

5. Find both a mean and a third proportional to two straight lines 1·2 and 1·8 in. long. Measure and write down their lengths.

6. Describe a regular octagon about a circle of $1\frac{3}{4}$ in. radius.

7. Bisect a straight line A B, 3 in. long, in C; through C draw C D 2·4 in. long, making the angle D C B 65°. Join A D and D B, measure and write down their lengths and the sizes of the angles of the triangle A D B.

8. Divide a straight line $3\frac{1}{3}$ in. long into 8 equal parts, and through the extremities of the line and through the points of division draw parallel straight lines, alternately continuous, dotted, and chain dotted, $2\frac{7}{8}$ in. long, and making angles of 40° with the first line.

9. Construct a triangle the sides of which measure 2, 2·6, and 3·1 in., and to a straight line, 1·7 in. long, apply a parallelogram of equal area, having one of its angles 58°.

10. Draw the segment of a circle to contain an angle of 46°, the chord measuring 1·7 in. Measure and write down the length of the radius of the circle.

11. To a circle of 1·8 in. radius apply an inscribed pentagon and circumscribed hexagon.

12. Draw a square of 3·8 square inches area and an equilateral triangle equal to it.

II.

1. At one extremity, A, of a straight line A B, $4\frac{1}{8}$ inches long, erect (without producing A B) a perpendicular A C $3\frac{1}{6}$ inches long; at the other extremity B, and on the same side of A B, draw B D $4\frac{1}{3}$ inches long, making the angle A B D 118°. Join CD, and divide it by construction into five equal parts.

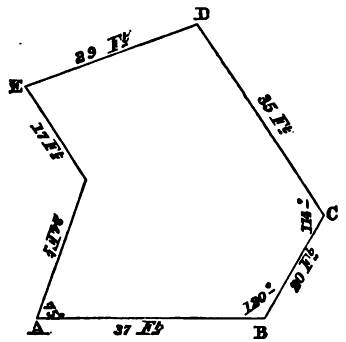
2. Determine by construction a mean proportional to two straight lines $1\frac{1}{8}$ and 2 inches long, and a fourth proportional to the same 2 lines and the mean proportional. Write down the lengths of both proportionals.

3. In a circle of 2·28 inches radius inscribe a regular heptagon, showing how its side is obtained; construct a square of equal area with the heptagon.

4. Construct the irregular figure of which the dimensions, &c., are given in the accompanying rough sketch.

Reduce the figure to a triangle of equal area having its vertex in D, and its base in A B (produced if necessary). Write down the magnitude of the angles of the triangle, and the length (in feet) of its sides.

Fig. 28.



Scale (which need not be drawn) 15 ft.=1 inch.

5. Draw three circles with radii of $\frac{3}{4}$, $1\frac{1}{4}$, and $1\frac{3}{4}$ inches, to touch one another externally.

6. Construct half an ellipse of which the major and minor axes measure 4·8 and 3·2 inches; at least 10 points in the circumference must be determined.

7. Draw a scale of 25 inches to the mile to measure yards; the smallest division on the scale is to represent 10 yards.

8. Draw a diagonal scale to measure miles, furlongs and chains; 50 furlongs being represented by 9 inches. From the scale draw a line 2 miles, 5 furlongs, 7 chains long.

N.B.—1 mile=8 furlongs=80 chains.

III.

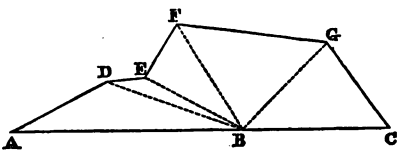
1. Upon a straight line 4·3 in. long construct a square, obtaining each perpendicular by a different geometrical process.

2. Describe a segment of a circle to contain an angle of 79°, the chord of the arc being $3\frac{7}{8}$ inches long. In the segment draw a circle with a radius of ·9 inches to touch both the chord and arc of the segment.

3. Construct the irregular figure of which the dimensions, &c., are given in the accompanying sketch.

Reduce the figure to a triangle of equal area having F for its vertex, and its base in A C (produced if necessary).

FIG. 29.



Scale $7\frac{1}{2}$ ft.=1 inch.

A B=20 ft.; B C=16 ft.;
B D=14 ft.; B E=10 ft.;
B F=13 ft.; B G=12 ft.;
A B D=28°; D B E=13°;
E B F=21°; C B G=58°.

4. Construct a triangle of which two sides measure 3·2 inches and 2·6 inches respectively, and the included angle 108°. Draw an equilateral triangle of equal area.

5. Divide a straight line $3\frac{3}{4}$ in. long in extreme and mean ratio. Find a third proportional (the less) to the two segments of the line, and write down its length.

6. With a radius of 2·35 inches, describe a circle, and divide it into five equal parts, each having the same area.

7. Draw a scale of feet and inches 6 feet long, in which 9 feet are represented by 10 inches. Show the calculation.

8. Draw a diagonal scale to measure perches, yards, and feet, 30 yards being represented by 7 inches. From the scale draw a line 3 perches, 2 yards, 1 foot long. Show the calculation.

N.B.—1 perch=$5\frac{1}{2}$ yards.

IV.

1. From a point A draw the straight lines A B, A C, A D, 2 inches, 3 inches, and 4 inches long respectively, making with each other equal angles (B A D, C A D, B A C) of 120° each. Join B C, B D, C D. Measure and write down the lengths of these lines and the size of the angles D B C, B C D, B D C.

2. Construct two squares on sides of $2\frac{1}{2}$ inches and $3\frac{1}{4}$ inches, and also a third square having an area equal to that of the sum of the other two. Erect three of the perpendiculars by different methods.

3. In a regular pentagon of 2·6 inches side inscribe a circle.

4. Construct a plain scale in which 564 feet are represented by 6·4 inches. The greatest dimension shown is to be 600 feet and the least 10 feet.

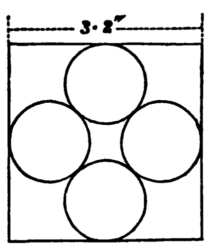
5. From the above scale construct an isosceles triangle, having a vertical angle of 85° and a base 330 feet long. Write down the length in feet of the equal sides, and the size of the equal angles.

6. On a line $3\frac{5}{8}$ inches long, construct the segment of a circle to contain an angle of 125°. Draw (in the segment) the lines containing the angle.

V.

1. Construct a scale of $\frac{1}{3750}$ to show 1,000 paces. A pace =30 inches. The scale to be properly figured.

FIG. 30.



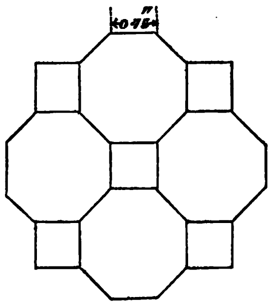
2. In a square of 3·2″ side, inscribe four equal circles touching each other and the middle points of the sides of the square as shown in the figure.

3. Divide a line 3·3″ long into 17 equal parts by construction. Divide the same line in the ratio 4 : 9.

4. Construct an isosceles triangle, base 2″, vertical angle 37°, and obtain a similar triangle of half the area.

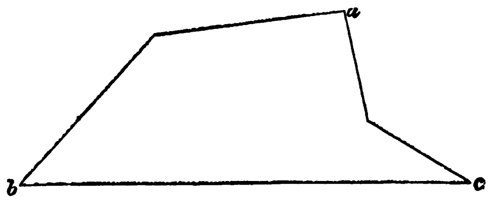
5. Draw the geometrical pattern shown in the figure. Side of octagons and squares 0·75″.

Fig. 31.



6. Draw the five-sided figure double the given size. Reduce it to a triangle with vertex at *a* and base in *b c* produced.

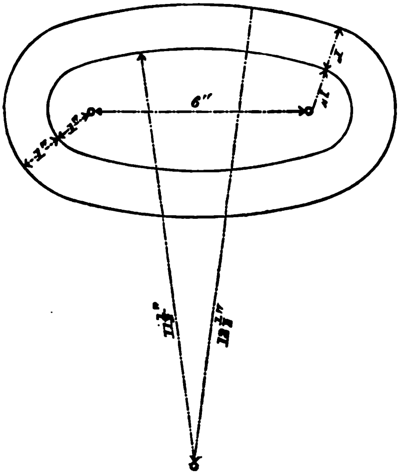
Fig. 32.



7. The given figure (fig. 33) represents one link of a chain. Its outline is made up of circular arcs, and all necessary dimensions are given. Draw the link to a scale of $\frac{1}{2}$ full size.

N.B.—It will be necessary to make the drawing near the middle of the paper in order that the centres of the larger arcs may not fall outside.

Fig. 33.



8. 'The angles in the same segment of a circle are equal.' Make use of this theorem in drawing a circular arc to pass through

Fig. 34.

the three given points *a*, *b*, *c*. (Fig. 34.) The centre of the arc is *not* to be employed.

VI.

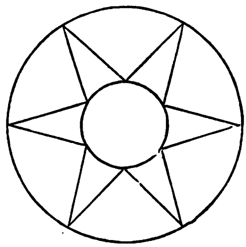
1. Construct a scale on which 80 feet are represented by 6 inches. Make the whole scale of such a length that 50 feet are shown by it.

2. Describe a regular pentagon of 2·25″ side, and reduce it to a triangle of equal area.

3. Draw a diagonal scale of 120 feet to an inch to measure single feet. Show 700 feet.

4. Draw the geometrical pattern shown in the figure. Radius of larger circle 1·75″, and that of smaller 0·65″.

FIG. 35.

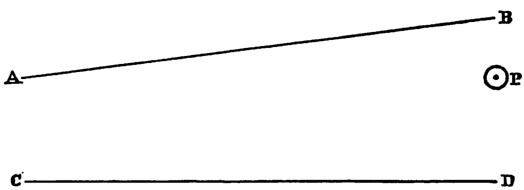


5. On a base 4·7″ long describe a triangle having its two sides 3·5″ and 3·9″, and in the triangle inscribe a square.

6. In a circle 1·75″ radius inscribe three equal circles touching each other and the given circle.

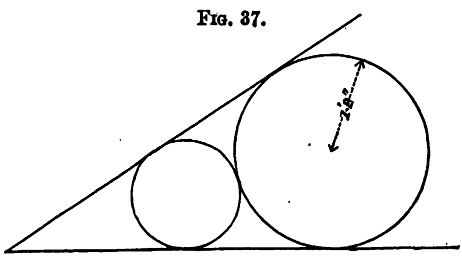
7. Draw A B and C D (fig. 36) two straight lines not parallel, but inclined at such an angle that the point of intersection cannot

FIG. 36.



be determined on the paper. Through a point P, not equidistant from B and D, draw a straight line that would if produced pass through the point of intersection of the two given lines.

FIG. 37.



8. Draw two straight lines containing an angle of 35°. With a radius of 1·2″ describe a circle touching both the lines containing the angle. Describe a second circle touching the first and the lines containing the angle. (Fig. 37.)

VII.

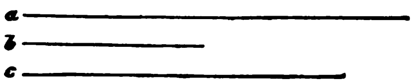
1. What is the representative fraction of a scale in which 210 yards are represented by 1·75 inches?

a. Construct a scale of 13 yards to 1 inch.

b. Construct a diagonal scale of $\frac{1}{570240}$ reading furlongs.

2. Determine a *fourth* proportional to the given lines *a*, *b*, *c*, and a *third* proportional to *a* and *b*. Write down the lengths obtained.

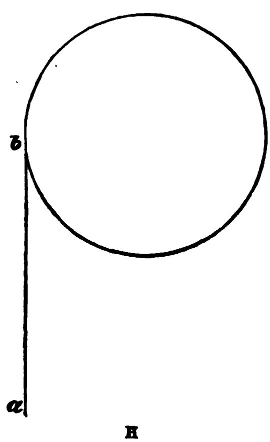
FIG. 38.



3. Show how a tangent can be drawn to a circle from any exterior point without making use of the centre of the circle.

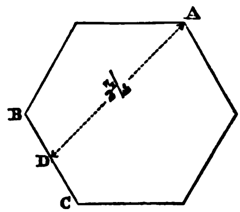
4. The line *a b* represents a piece of thread unwound from the given circle. Draw the curve traced by the extremity *a* when the thread is wound back on to the circle.

FIG. 39.



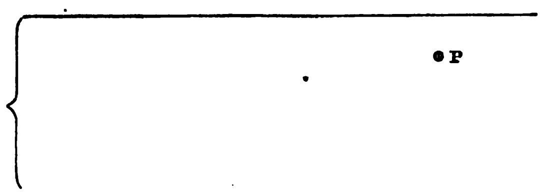
5. In a hexagon the distance from an angle A to D the middle point of the side $B\ C$ is $2\frac{1}{4}''$. Draw the hexagon.

FIG. 40.



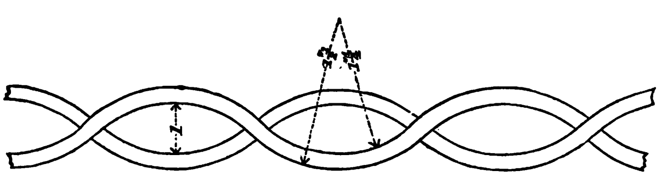
6. Draw a circle touching the given parallel lines and passing through the given point P. (Fig. 41.)

FIG. 41.



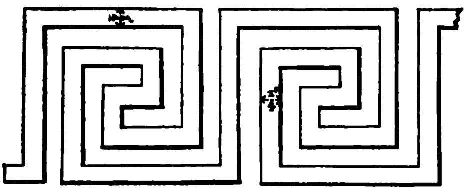
7. The figure shown (fig. 42) is made up of arcs of circles of two different radii. Sufficient dimensions are given; draw the figure.

FIG. 42.



8. Draw the 'Greek fret' or 'key pattern' shown in the figure (fig. 43), the lines to be all $\frac{1}{4}''$ apart. The distinction between fine and strong lines is to be preserved.

Fig. 43.



VIII.

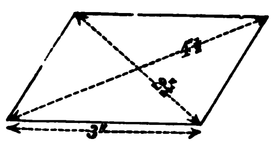
1. Construct a scale of $\frac{1}{844800}$ to show 80 miles. Construct also a comparative scale showing 100 versts (a verst equals 693·3 yards).

2. Draw six parallel straight lines 4″ long 0·45″ apart. Ink in these lines, gradually increasing the strength from the highest to the lowest. Crossing these lines at right angles, draw a series of nine dotted lines 0·5″ apart.

3. Draw a series of circles, diameters, 1″, 1·25″, 1·5″, 1·75″, touching each other successively and all touching a given line.

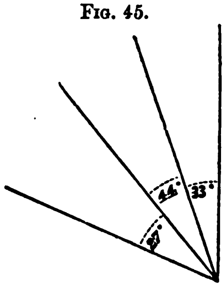
*4. Construct a parallelogram from the given dimensions.

Fig. 44.



*5. By means of the protractor, plot the given angles in the order shown. In each angle inscribe a circle of 1½″ *diameter.*

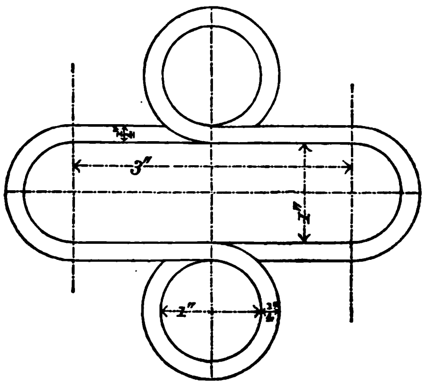
Fig. 45.



6. Determine a mean proportional between two lines 2·35″ and 1·25″. By a similar construction obtain $\sqrt{3}$, taking 1″ as unit.

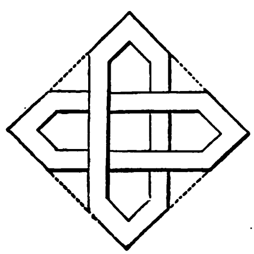
*7. Draw the given figure from the dimensions shown.

FIG. 46.



*8. Draw the given geometrical figure twice the size shown. The distinction between fine and strong lines to be preserved.

FIG. 47.



R.I.C.E. College, Cooper's Hill Papers.

I.

1. Construct a scale to read feet and inches, $8\frac{1}{2}$ feet being equivalent to 5·27 in. Mark the representative fraction.

2. Draw the segment of a circle with a radius of $1\frac{3}{8}$ in. to contain an angle of 36°; and draw two other circles having radii of ·72 in. and ·95 in. touching each other, and the arc of the segment externally.

3. At one extremity of a straight line 2·4 in. long, erect a perpendicular $1\frac{1}{8}$ in. long, without producing the line. Inscribe a circle in the triangle formed by the perpendiculars, and the line joining their extremities.

4. Draw a circle having an area of 6 square in., and divide its area by means of concentric circles into 4 equal areas. (The solution must be entirely geometrical, π being assumed 3·14.)

5. Construct a diagonal scale of $\frac{1}{500}$ to measure mètres, decimètres (tenths of mètres), and centimètres (hundredths of mètres), the mètre measuring 1·0936 English yards.

6. Draw a regular heptagon of 1·4 in. side, and reduce it to a triangle of equal area.

7. Determine a fourth proportional to three straight lines of the respective lengths of 1, $1\frac{3}{8}$, and $1\frac{15}{16}$ in.

8. Construct a square and an equilateral triangle each having an area of 3·8 square in. (The solution must be entirely geometrical.)

II.

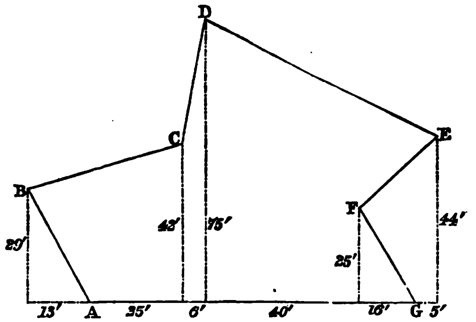
1. From the extremities of A B 3·3 inches long, and on opposite sides of it, draw A C, B D, $2\frac{1}{4}''$ and $3\frac{5}{8}''$ long respectively, making the angles C A B, D B A 113° and 81° respectively. Join C D, and divide it by construction into 5 equal parts.

2. Construct a square on a side $3\frac{1}{4}''$. In this square inscribe a circle; and in this circle inscribe 3 equal circles.

3. Construct from the marked dimensions the figure A B C D E F G, of which the annexed diagram is a rough copy. Reduce the figure to a triangle of equal area, having its vertex in D and its base in A G (produced if necessary). Measure the sides in

feet, and the angles of this triangle and write down their magnitudes. Scale 25 feet = 1 inch.

FIG. 48.



4. Draw a straight line 6 inches long and mark a point an inch perpendicularly above its centre; draw the arc of a circle to pass through this point and the extremities of the line; at least four points in the arc must be determined.

5. Construct a circle and equilateral triangle, each to contain an area of 7·5 square inches. The processes employed must be purely geometrical.

6. Draw a scale of feet to measure all distances between 70 feet and 1 foot, $5\frac{1}{2}$ feet being represented by ·52″; by the method of diagonal division make this scale available for measuring inches. From the scale draw a line 45′·2″ long.

7. The horizontal traces of two planes, having inclinations of 35° and 55° respectively, intersect at an angle of 60°. Determine the intersection of the planes, its inclination to the horizon, and the angle contained between the planes.

8. A tetrahedron (i.e. a solid bounded by 4 equal and equilateral triangles) of 3 inches side, has one of its edges horizontal and an adjacent edge inclined at 40°. Draw a plan of the tetrahedron.

III.

1. Upon a straight line 3·15 inches long, and upon the same side of it, describe two segments of circles containing respectively 145° and 65°. Draw the angle in each segment.

2. Draw three circles having radii of 1, $1\frac{1}{2}$ and 2 inches respectively to touch each other externally.

3. On a straight line $2\frac{1}{2}$ inches long, and upon the same side of it, construct an equilateral triangle and a regular pentagon. Reduce the space included between the outside of the triangle and the inside of the pentagon to a triangle of equal area.

4. Draw a straight line A B 98 feet long; on it mark points C D E F distant respectively 18, 41, 62, and 77 feet from A; at C and D erect perpendiculars C G, D H, on one side of A B, 22 and 28 feet long; and at E F erect perpendiculars E K, F L on the opposite side of A B, 35 and 32 feet long. Join A G, G H, H K, K L, L B. Write down the lengths (in feet) of these lines and the magnitude of the angles A G H, G H K, H K L, and K L B. Scale, 15 feet = 1 inch.

5. Determine geometrically the values of $\sqrt{3}$ and $\sqrt[3]{2}$.

6. The relative areas of a map and the country it represents are 121 square inches and 144 square miles. Draw the scale of the map to show miles and furlongs, and diagonally chains.

From the scale draw a line 3 miles 2 furlongs 6 chains long. Mark the representative fraction of the scale.

7. Two lines 3 and 4 inches long and inclined to the horizon at 25° and 35°, bisect each other at right angles. Draw the plan of the lines and the traces or scale of slope of the plane containing them. Determine the inclination of this plane to the horizon.

8. Draw the plan of a cube of 3 inches edge when one face is inclined at 40° to the horizon and an edge in that face of 20°.

IV.

1. Construct an isosceles triangle with a base of $4\frac{1}{4}$ inches and a vertical angle of 65°. Construct a square of equal area with the triangle.

2. Describe a circle with a radius of 2·58 inches and by means of 4 concentric circles divide its area into 5 equal parts. From any point in the outermost circle draw a tangent to the innermost circle.

3. Construct a scale, in which 5 furlongs are represented by ·9 inches, to show miles and furlongs, and (diagonally) chains; mark the representative fraction. The scale should be long enough to measure 5 miles. Show the calculation.

4. Seven straight lines making equal angles ($\frac{360}{7}^\circ$) with each other radiate from a common centre. Make these lines respectively, taken in order, 60, 50, 40, 30, 20, 55, and 25 feet long. Join their extremities. Reduce the irregular figure of seven sides thus formed to a triangle of equal area. Write down the length (in feet) of each of the three sides, and the size of each of the three angles of the triangle. Scale (which need not be drawn) $25' = 1$ inch.

5. In a circle $2\frac{1}{4}$ inches radius inscribe a regular pentagon, and in the pentagon inscribe an equilateral triangle.

6. Determine geometrically the values of $\sqrt{5}$ and of $\frac{\sqrt{5}}{\sqrt{3}}$.

7. The plan of an equilateral triangle of 3 inches side is a triangle having two of its sides $2\frac{1}{2}$ and $1\frac{1}{2}$ inches, and the included angle 75°. Determine the scale of slope (or the traces) of the plane in which the triangle lies, and the inclination of this plane and the third side of the triangle to the horizon.

8. Draw the plan of a right hexagonal prism 4 inches long, of which each side of the base measures $1\frac{1}{2}$ inch, supposing one side of the base to rest on the horizontal plane and a face containing that side to be inclined 50° to the horizon.

V.

1. Upon a base B C, 2 inches long, describe a triangle A B C having the side A C 1·75 inches and the angle A B C equal to 55°. Upon A B construct a square A B E F, and on A C construct a square A C G H. Join F H, and reduce the whole figure F E B C G H to a triangle of equal area.

2. The distance between 2 places is $7\frac{1}{2}$ miles, and measures on a map 1·1 inches. Draw a scale of miles to suit the map representing 40 miles. By the diagonal method make the scale to show furlongs. Write down the representative fraction, and show your calculations.

3. With a centre C and radius of 3 inches describe a sector of a circle A C B, having the angle at C equal to 60°. In the sector inscribe a square having two of its corners in the arc A B.

4. With a centre C and radius C B equal to 1·5 inches describe a circle and set off an angle B C A equal to 45°. Make C A equal to 2·75 inches and describe a second circle touching the first in B and passing through the point A.

5. Describe a triangle having its three sides 2·75 inches, 2·25 inches, and 1·5 inches respectively, and divide the triangle into three parts in the ratio of the numbers 3, 4, and 5 by lines parallel to its shortest side.

6. Determine the plan and elevation of a cube of 2 inches edge, having one face inclined at 50° and one edge in that face making an angle of 25° with the Horizontal Plane.

APPENDIX.

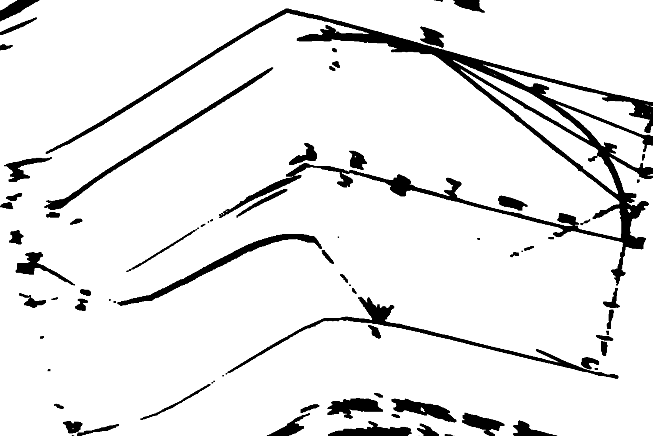
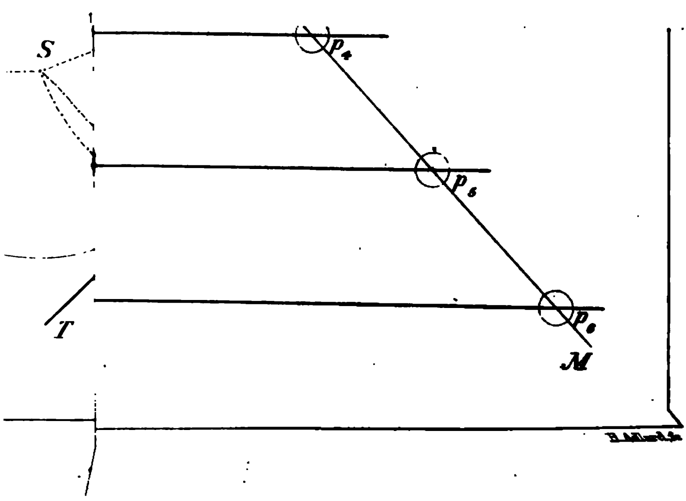


PLATE II.

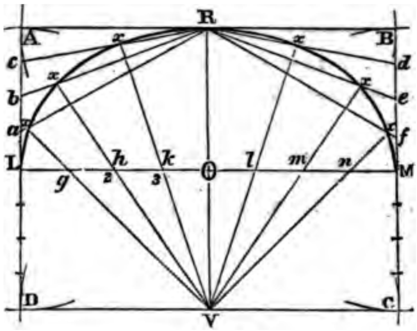


APPENDIX.

Problem.—To describe an ellipse, having given the axes.

Let L M and R V be the axes intersecting in the centre O. With centres R and V and a radius equal to O L, semi-axis major, describe arcs at A, B, C, D. With centres L and M and a radius O R, equal to semi-axis minor, describe arcs cutting former arcs in A, B, C, D. Join A B, B C, C D, D A. Divide O L, O M, C M, M B, D L, L A,

FIG. 49.



each into the same number of equal parts, say four, draw lines *a* R, *c* R, *d* R, *e* R, *f* R. Join V G, produce it to meet *a* R in *x*; join V *h*, produce it to meet *b* R in *x*; join V *k*, produce it to meet *c* R in *x*; join V *l*, produce it to meet *d* R in *x*; join V *m*, produce it to meet *e* R in *x*; join V *n*, produce it to meet R *f* in *x*. There are thus obtained six points in curve of ellipse, and, by a similar process, the under half may be obtained. The ellipse must be drawn carefully by hand.

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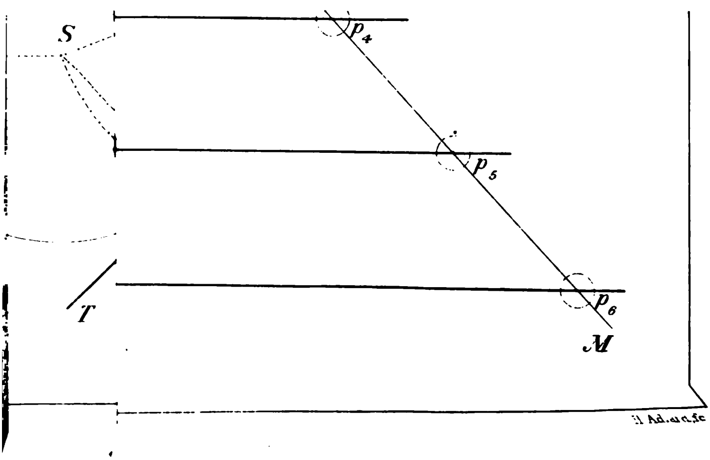

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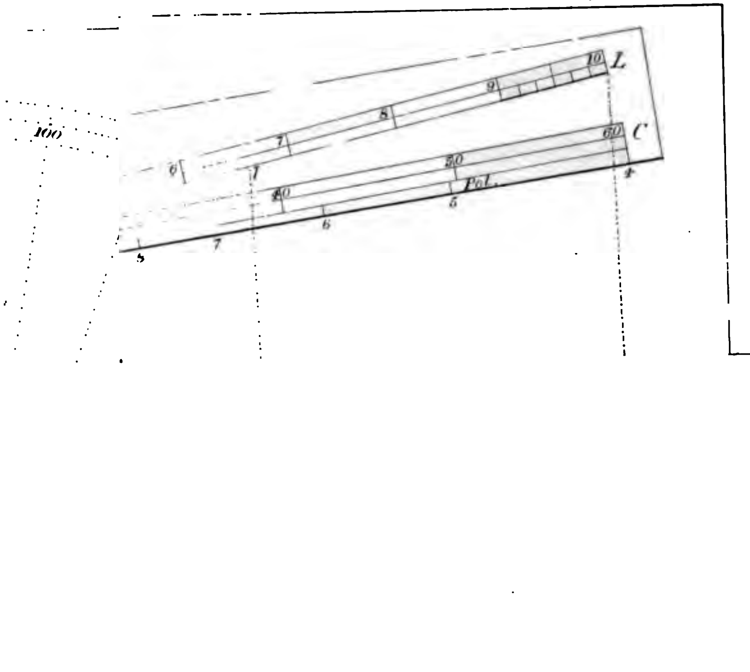

H. Adlard, Sc.

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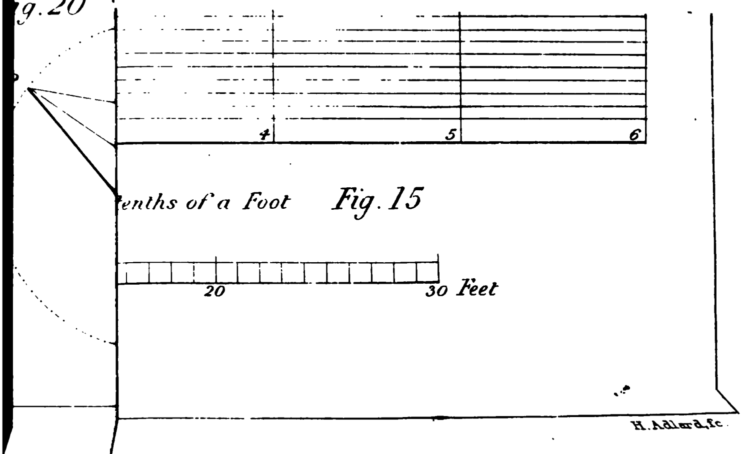

enths of a Foot

Fig. 15

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